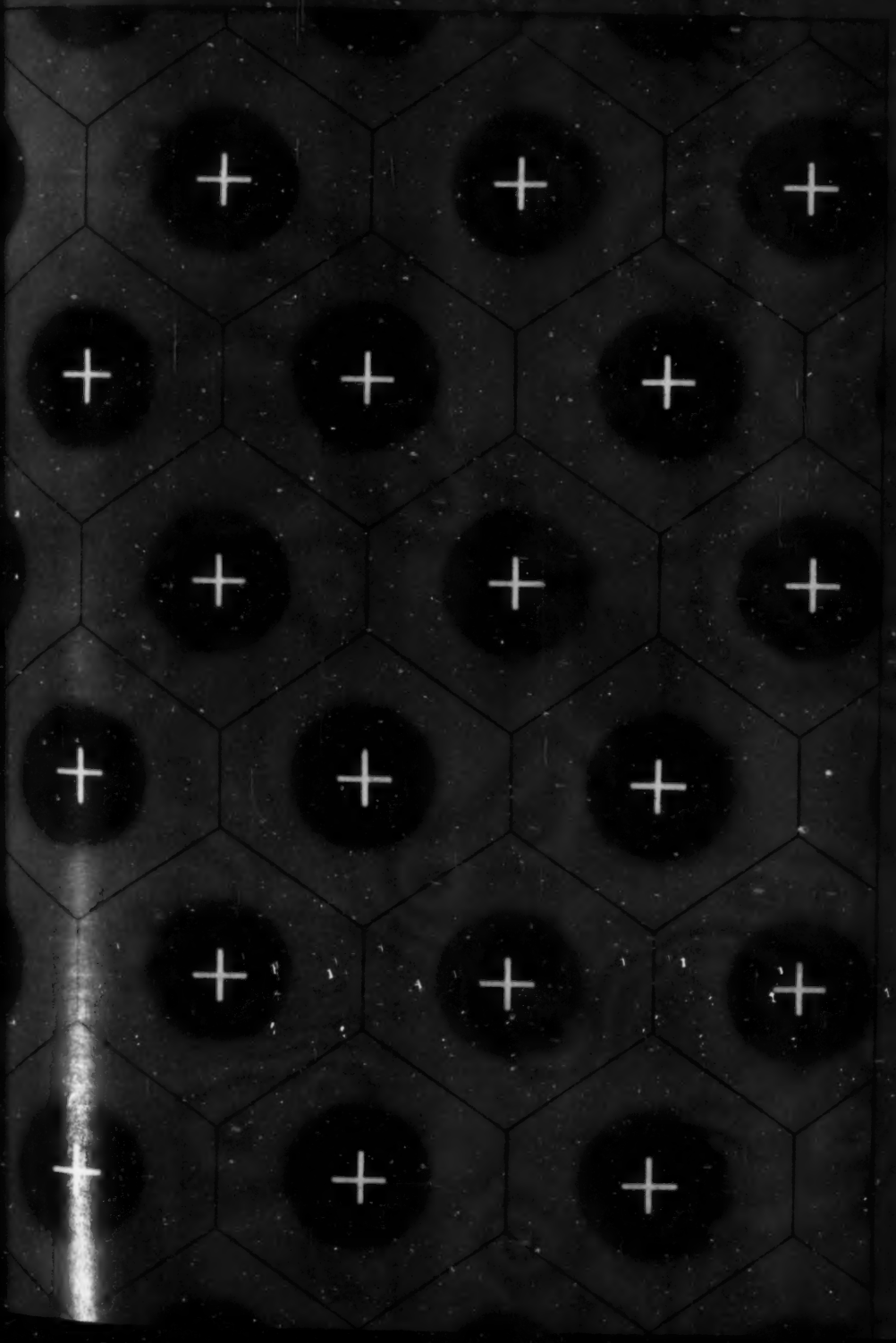
JOURNAL OF APPLIED PHYSICS



AUGUST
19[illegible]
Vol. [illegible] No. 8

Nature of the Metallic State

W[illegible] SHOCKLEY

JOURNAL OF APPLIED

PHYSICS

AUGUST, 1939 Vol. 10, No. 8

In This Issue

Page

Shall We Kill the Goose that Lays the Golden Eggs? 525

Sound Measurements in Industry ERNEST J. ABBOTT 526

Resumés of Recent Research 532

Applied Physics in the Bureau of Home Economics MARGARET B. HAYS 537

Nature of the Metallic State WILLIAM SHOCKLEY 543

Accessories for Spectrochemical Analysis STANLEY S. BALLARD AND PAUL L. GOW 556

Contributed Original Research

X-Ray Investigation of Crystallinity in Rubber S. D. GEHMAN AND J. E. FIELD 564

Effects of Irradiation, Humidity and Sphere Material on the Sparkover Voltage of the Two-Centimeter Sphere Gap ARTHUR B. LEWIS 573

Thermal Distribution and Temperature Gradient in the Arc Welding of Oil Well Casing W. A. BRUCE 578

Convection and Conduction of Heat in Gases I. BRODY AND F. KŐRÖSY 584

In Every Issue

Here and There 558

New Books 559

Contributors to This Issue 562

Innovations in Instruments 563

Index to Advertisers viii

BOARD OF EDITORS

ELMER HUTCHISSON, *Editor*

Associate Editors

F. BITTER
W. E. FORSYTHE
L. A. JONES
W. F. MEGGERS
B. E. WARREN
J. B. WHITEHEAD
V. K. ZWORYKIN

Publication of the **American Institute of Physics**

ADMINISTRATIVE STAFF

HENRY A. BARTON, *Director*

MADELINE M. MITCHELL, *Publications Manager*

F. K. RICHTMYER *Chairman, Advertising Committee*

GOVERNING BOARD

JOHN T. TATE, *Chairman*
KARL T. COMPTON
E. C. CRITTENDEN
WHEELER P. DAVEY
FLOYD A. FIRESTONE
HARVEY FLETCHER
ARTHUR C. HARDY
PAUL E. KLOPSTEG
MELVIN MOONEY
FREDERIC PALMER, JR.
R. L. PEEK, JR.
GEORGE B. PEGRAM
W. B. RAYTON
F. K. RICHTMYER
WALLACE WATERFALL

THE JOURNAL OF APPLIED PHYSICS, published monthly at Prince and Lemon Sts., Lancaster, Pa., is devoted to physics in its role as the science basic to other natural sciences and to the arts and industries. Previous to 1937 this publication was known as PHYSICS. It aims to be of service not only in physical laboratories, but also in the laboratories of industrial, chemical, geological, geophysical, meteorological, radio, and similar concerns. THE JOURNAL OF APPLIED PHYSICS publishes editorials, news of physicists and reviews, as well as technical papers of applied physics.

SUBSCRIPTION PRICE

	U. S., POSS. AND CANADA	FOREIGN
To members of scientific societies	$5.00	$5.70
To all others	7.00	7.70

Manuscripts should be submitted to Elmer Hutchisson, Editor, University of Pittsburgh, Pittsburgh, Pennsylvania.

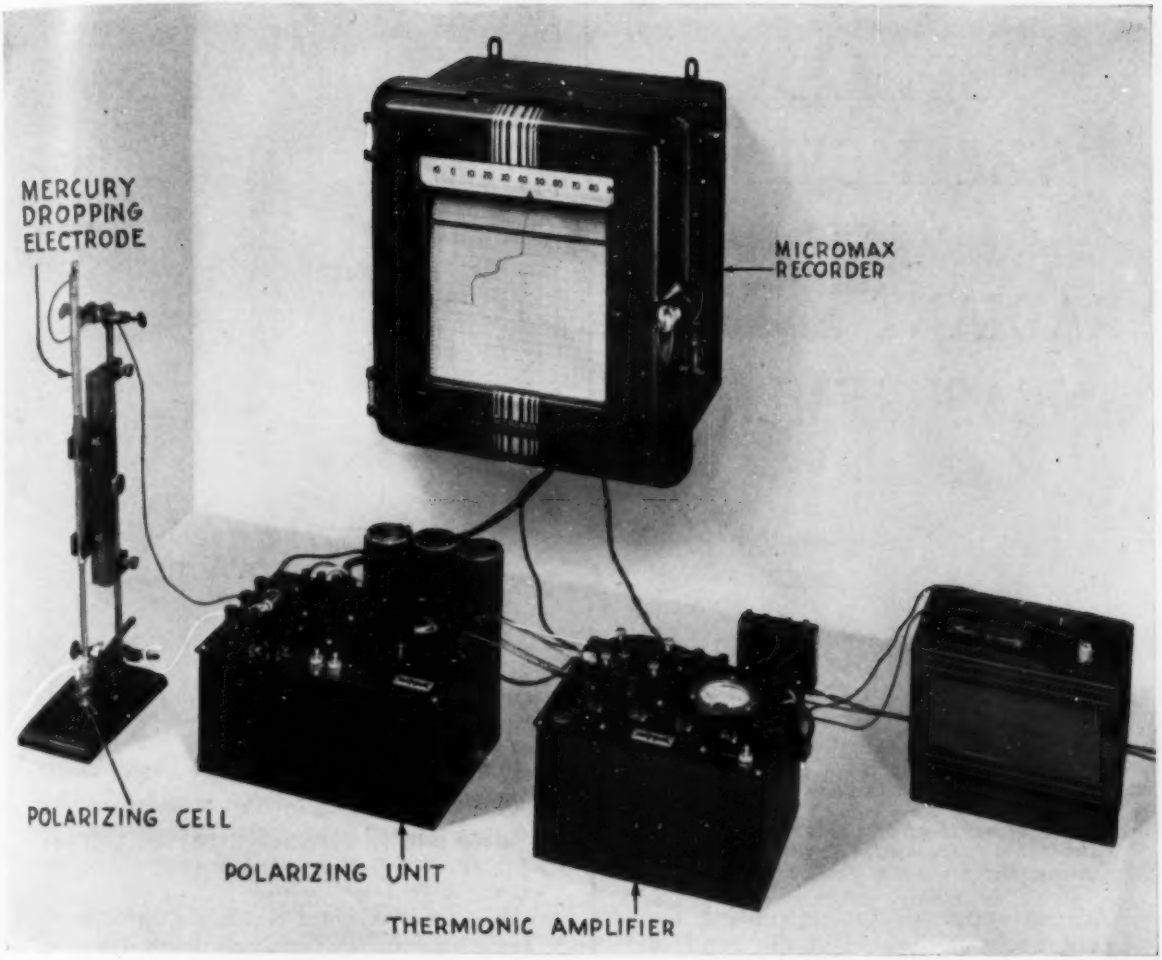
Changes of address, proofs, and complaints of failure to receive the JOURNAL OF APPLIED PHYSICS should be addressed to the Publications Manager. New copies can be sent free only if notice of nondelivery is received within three months of date of issue.

Subscriptions and orders for back numbers should be addressed to Prince and Lemon Streets, Lancaster, Pa. or to the American Institute of Physics, 175 Fifth Avenue, New York, N. Y.

Advertising rates supplied on request. Orders, advertising copy and cuts should be sent to the American Institute of Physics.

Entered as second class matter at the Post Office at Lancaster, Pennsylvania, under the Act of March 3, 1897. Accepted for mailing at the special rate of postage provided for in the Act of February 28, 1925, authorized May 2, 1932.

Copyright 1939, by the American Institute of Physics.



New! Qualitative and Quantitative Analyses From Micromax Current/Potential Recorder

Using the Dropping Mercury Electrode principle, this new "Electro-Chemograph" automatically inks a current-potential curve on the wide Micromax chart. From this easily-read "Electro-Chemogram," chemical analyses for many elements are quickly derived.

An assembly of standard L&N units, the equipment is reliable and accurate. Outstanding advantages of the L&N equipment include: 1. The operator sees the current/potential curve being inked. 2. The wide chart is easily and accurately read, and is permanent. 3. Dark rooms, developing processes and delicate galvanometer systems are eliminated. 4. Instruments are made up of standard L&N mechanisms, proved reliable over years of use in other applications.

For its most successful use, a detailed study of the specific application must be made by the user. For the present, research laboratories establishing routine procedures will probably be its principal users. After these procedures have been worked out, the Electro-Chemograph will provide a fast, convenient method for routine testing.

WRITE FOR TECHNICAL PUBLICATION E-94(1)

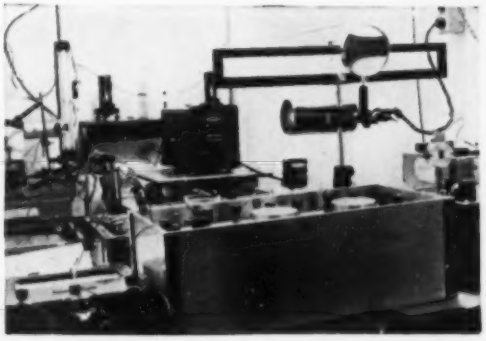
LEEDS & NORTHRUP COMPANY, 4978 STENTON AVE., PHILA., PA.

LEEDS & NORTHRUP

MEASURING INSTRUMENTS • TELEMETERS • AUTOMATIC CONTROLS • HEAT-TREATING FURNACES

JrlAdE-3(1)

EPPLEY THERMOPILES *for* RADIANT ENERGY MEASUREMENTS



A view of some of the apparatus used for calibrating thermopiles at the Eppley Laboratory, Inc.

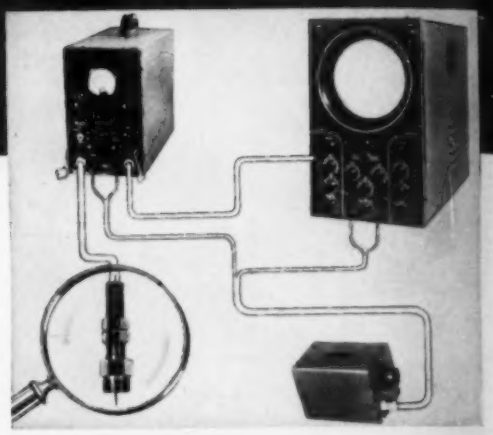
Adequate manufacturing, testing and calibrating equipment in the hands of our skilled and experienced technicians insures satisfaction to the users of Eppley Thermopiles.

We are prepared to furnish thermopiles of either bismuth-silver or copper-constantan, mounted in vacuum or air type cases. We also furnish thermopiles and mountings especially designed to meet individual needs. A card will bring you our Bulletin R–3 which contains illustrations, sample curves, complete descriptive matter, prices, references, etc., with regard to Eppley Thermopiles.

EPLAB

THE EPPLEY LABORATORY, INC.
SCIENTIFIC INSTRUMENTS
NEWPORT, R.I.
U.S.A.

Here's the RCA Electric Engine-Indicator Unit!



Make accurate and dependable measurements of rapidly changing cylinder pressures in high speed internal combustion engines with this RCA equipment

This newly designed electrically operated engine-indicator diagram equipment has been developed by RCA engineers to measure all dynamic pressures. It eliminates the inaccuracies of mechanical indicating instruments which are caused by friction and inertia and assures instant sensitivity, easy operation.

This RCA equipment is composed of four units—a pressure unit of revolutionary design, a 9-inch cathode ray oscillograph, an amplifier and a synchronizer unit. It provides high accuracy, covers many fields of application.

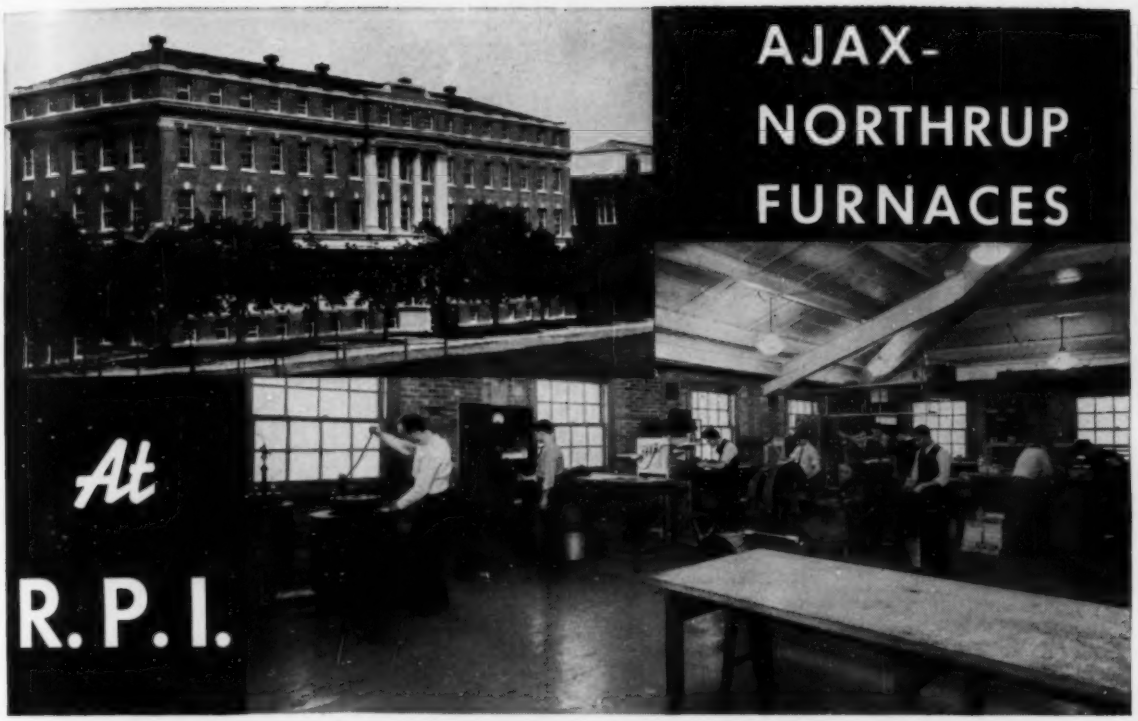
Use RCA radio tubes in your test instruments—for greater precision

Write for further details and description

RCA

Precision Test Equipment

RCA Manufacturing Company, Inc., Camden, N. J.
A Service of the Radio Corporation of America



"The Ajax-Northrup Furnace installed in the Metallurgical Laboratory at the Rensselaer Polytechnic Institute has been in continuous use by graduates and undergraduates since it was installed in 1924.

"It has been the melting tool used in most of the metallurgical theses which have been presented for graduation.

Metallurgical Laboratory, Rensselaer Polytechnic Institute, and general view of the top floor. Every undergraduate in the chemical and metallurgical courses for the past 15 years has used and become familiar with the Ajax-Northrup Furnace.

"It has been used continuously by graduate students and members of the staff for the following:

"For making a wide variety of alloys with useful physical and electrical properties.

"For investigations in the field of magnetic alloys, particularly of iron, nickel and aluminum.

"For melting in vacuum where oxidation during melting was a factor to be avoided.

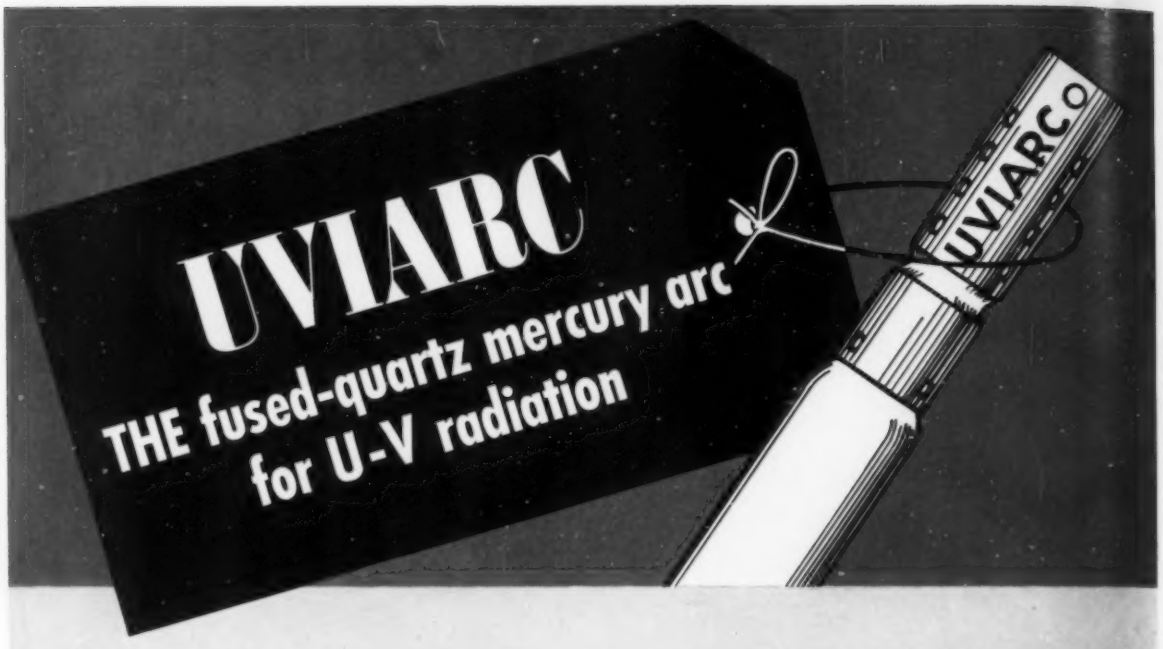
"As a sintering tool in powder metallurgy.

"As a melting tool for powdered materials such as refractory carbides which could not be melted by conventional means.

"We know that this furnace will continue to be of service in a wide variety of fields."

AJAX ELECTROTHERMIC CORPORATION
•
AJAX PARK
TRENTON, N. J.

AJ-14-G



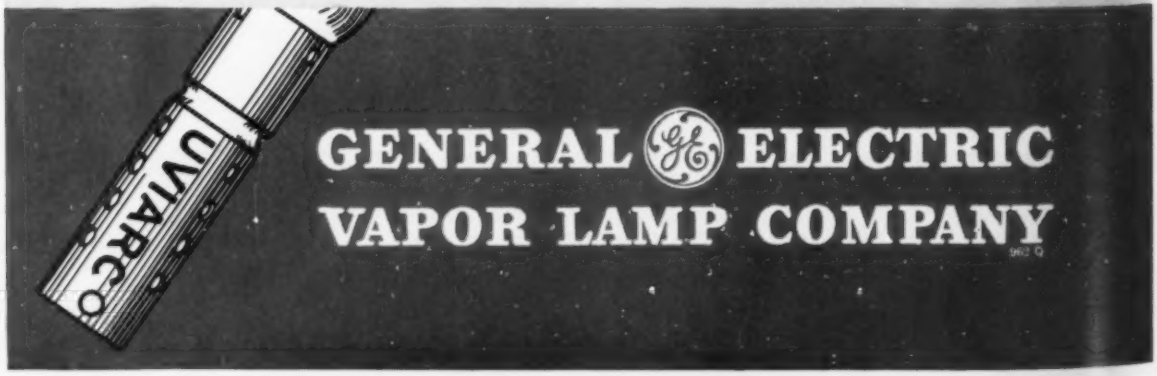
UVIARC is the trade mark for the General Electric fused-quartz mercury arc – a highly efficient source of the complete mercury vapor spectrum, and in particular, of ultra-violet radiation.

ADAPTABLE – UVIARC tubes can be built into the user's own apparatus to form economical and convenient sources for college, high school and experimental laboratories. The outfit is easily wired, and auxiliary equipment required consists only of a transformer and a compact starter assembly.

SELF-STARTING – UVIARC tubes operate on alternating current; they are self-starting and can be used in any operating position at full efficiency.

FLEXIBLE – UVIARC tubes may be obtained in convenient wattages and lengths, ranging from 150 to 1,200 watts, and from 1⅜ in. to 12½ in. effective length. Higher wattages and longer lengths available for special applications.

RADIATION DATA – UVIARC tubes are completely described in a recently published paper by Johnson and Webster, "Important Improvements in Fused-Quartz Mercury Arcs." Reprints and further information are available from General Electric Vapor Lamp Company, 835 Adams Street, Hoboken, New Jersey.



HARVARD COLLEGE
AUG 2 1939
LIBRARY

Journal of Applied

Physics

Volume 10 August, 1939 Number 8

Shall We Kill the Goose that Lays the Golden Eggs?

RECENTLY an editorial in one of the metallurgical periodicals congratulated physicists on a "movement within their own ranks" to bring physicists down to earth. It was said that inbreeding, overproduction and gross neglect of everyday physics has resulted from an overemphasis of the quantum point of view and high-brow speculations on atomic structure to the exclusion of more tangible problems. Unfortunately, we must admit some justification for this criticism. In this Journal there has been a continuous plea for more attention to those borderline problems in physics which have immediate application in industry and in other sciences. Yet one should be wary of criticizing physicists for allowing themselves to go on extending knowledge in those directions in which quantum mechanics applies even if at the present time industry can't make use of their results.

We have all heard stories of the contemporary criticism of Faraday for his eternal tinkering with wires and magnets. Didn't even Maxwell make some "high-brow speculation"? Roentgen certainly spent a lot of time worrying about discharges in gases, and now every hospital and almost every large industry makes use of the rays which bear his name. One can guess that those who developed the laws of the emission of electrons from filaments were said to be working with their "heads in the clouds" and yet no single development in the last century has had as great effect on everyday life as that of the vacuum tube. In spite of the fact that electron diffraction comes as a direct result of the "quantum point of view," it has already become an engineering tool used for the study of surface properties.

When one really gets down to cases, are there many things which the physicist has discovered which have not been used by somebody to produce dividends? Quantum mechanics itself seems to be showing us how to understand alloys that have extraordinary physical properties. There is a serious question as to whether power cannot be developed from the splitting of the uranium nucleus by bombardment with neutrons. Would our nuclear physicists be called high-brow if they discovered a cheap fuel for heating our houses? We begin to wonder whether the physicist is altogether to blame. Maybe if those who criticize picked up the new discoveries of physics a little quicker there would be less "useless physics." To ask the physicist to stop extending frontiers even though they lead to quantum mechanics seems much like killing the goose that lays the golden eggs.

For September

Radium Protection, by EDITH QUIMBY

Economic Features of X-Ray Protection, by L. S. TAYLOR

Report of the Meeting of the Advisory Council on Applied Physics

Also contributed original research.

Sound Measurements in Industry

BY ERNEST J. ABBOTT

Physicists Research Company, Ann Arbor, Michigan

ACOUSTICS, in common with the other branches of physics, has been experiencing considerable activity of late. Basic work in architectural acoustics and communication began about the turn of the century, and the fundamentals of these fields are reasonably well in hand. The application of sound measurements in industry began about 1925 with the advent of the condenser microphone and the vacuum tube amplifier. The fundamentals of this field are not nearly as well developed.

Classification of Sound Measurements in Industry

Sound measurements in industry can be classified in about four groups: (1) Routine noise inspection, (2) noise specifications, (3) descriptive terminology for general use, and (4) Diagnosis of machinery noise as an aid to quieting.

On many items such as ball and roller bearings, gears, and domestic equipment, noise is an important factor in customer acceptance. Such items are often given a 100 percent noise inspection, and production held to surprisingly close limits. The drawbacks of ear testing have been recognized for years, and there has been a long standing demand for instrumental noise testing instruments.

In cases where noise is important, as for example in ventilating equipment, power transformers in residential locations, etc., numerical noise specifications are badly needed. Such specifications are valuable both to the supplier and to the purchaser. If there were some scale for expressing noise so that the prospective purchaser could describe his needs to the supplier, much confusion would be eliminated. Once the specifications were agreed upon, it would be an easy matter to tell if they were met. Many costly arguments and much dissatisfaction would be avoided in this way. The need for such specifications has been recognized for years.

We also need a general language for describing noise. The vendor of acoustical treatment needs some scale for describing to his prospect just what improvement will be achieved by acoustical treatment of his office, church, or factory. We need numbers for describing the noises which exist in various places, and the improvements which accompany proposed or considered changes.

Sound measurements find wide application in the diagnosis of noise problems, especially machinery noise. Since this is the particular field in which the author has labored, it will form the basis of this paper. The need for measurements in this work has long been recognized.

In the past 15 years many attempts have been made to use sound measurements in all of the four classifications listed above. Some outstanding successes have been achieved, but probably 90 percent of the attempts would be classified as commercial failures. Today, most noise inspections are made by ear, noise specifications are little used, vendors of acoustical material attempt to show a prospect a similar installation, and most attempts to quiet machinery are made on the basis of ear listening.

On the other hand, there are successful noise tests in everyday operation, noise specifications are being used with satisfaction to both supplier and purchaser, noises can be described in terms of measurements, and some outstanding jobs of machinery quieting have been accomplished with sound measurements. The successful jobs appear to be no more difficult than the unsuccessful ones, and the question immediately arises as to the reasons for the wide divergence in success.

The answer appears to be twofold: (1) Lack of suitable instrumentation, and (2) lack of appreciation of the fundamentals of the field.

From the start, the available instrumentation for industrial sound measurements has been woefully inadequate. In the past two years, the situation has improved somewhat, but most of the available instruments are still impractical for

everyday industrial noise work. The reasons lie in the fundamentals of the field, and accordingly the two will be discussed together. Some of the basic items will be considered individually.

Dynes per Square Centimeter versus Decibels

Almost all of the soundmeters in use today are essentially pressure measuring devices. The accepted unit for such measurements is the dyne per sq. cm. In my opinion, many of the past, present, and future difficulties of industrial sound measurements would have been avoided if the dyne per sq. cm were used for expressing all sound measurements. Instead, use is made of the decibel scale, which "begins nowhere, and extends to infinity in both directions." Certainly, decibels have their uses in telephony and similar fields, but they cause endless confusion in industrial sound measurements. (Of course, we use decibels in our laboratory. To do otherwise would be like using classic Greek in an American-speaking land. Besides, we think that *we* understand them—until the next time we get scrambled.)

It is hard to explain to an industrialist why it is that two men can and do measure the same sound and disagree by five or ten decibels. Apparently the decibel has different values in different cities.

Seriously, the decibel scale furnishes a most convenient means for covering up the fact that one does not really know the calibration of his equipment. If the dyne per sq. cm, were used the question of "reference level" would be brought out in the open, and we would soon have to all use the same kind of dynes.

It is often stated that "the ear acts on a decibel scale." This is misinformation, pure and simple. Fig. 1 shows measurements on loudness published by The Bell Telephone Laboratories, and thoroughly confirmed by other workers in this country and abroad. The highest level to which one can listen without pain is just about 1,000,000 times as loud as the faintest sound one can hear. Intermediate loudness ratios are systematically arranged between these extremes. As shown in Fig. 1, the data depart only moderately from a straight line, and the best straight line fit over the range is the sound pressure in dynes per sq. cm. For practical purposes, it is a close approximation to say that the loudness of a sound is doubled when the sound pressure is doubled, and halved when the sound pressure is halved.

In contrast to this, consider the decibel scale. The range of loudness which the ear hears as a million to one is compressed into a scale of 120 numbers. This leads to the practical absurdities that on a loudness basis, half of 110 is 101, half of 100 is 91, etc. No, the real reason for using decibels is convenience in plotting graphs. The ear has an important characteristic which it is impossible to show on a graph, namely, it can appreciate a *large* number of *large changes* in loudness. Such a thing is impossible to represent graphically. One can show a few large changes, or many small changes, but not many large changes. The decibel scale allows one to cover a large range, but makes the differences appear much smaller than they do to the ear.

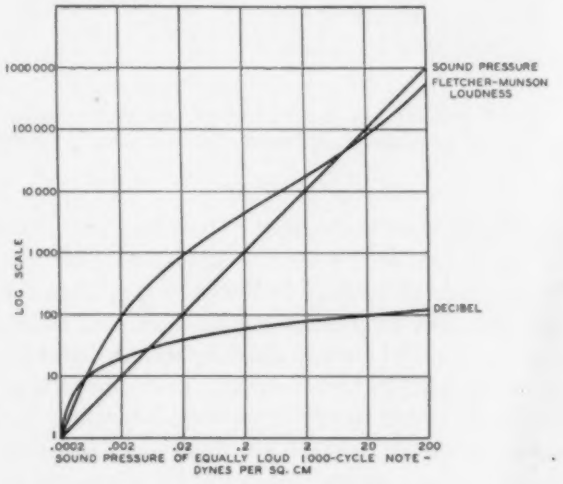


FIG. 1. Relation between sound pressure and loudness as determined at Bell Telephone Laboratories. Note that there is essentially a linear relation between loudness and sound pressure, although they are both plotted to logarithmic scales in order to cover the audible range. The decibel scale does not agree at all with the loudness scale.

If all decibel measurements were taken so the data could be immediately translated into absolute units such as the dyne per sq. cm., all would be well, but unfortunately such is not the case.

Sound Pressure versus Loudness

Many investigators have measured sound pressure, and have attempted to call the result loudness. That did not make it so. Others have used a

frequency weighting on their sound pressure measurements, and called that loudness. That was not so either. Probably the most vigorous and consistent objector to this practice has been Dr. Harvey Fletcher, of The Bell Telephone Laboratories, and there is ample proof that his

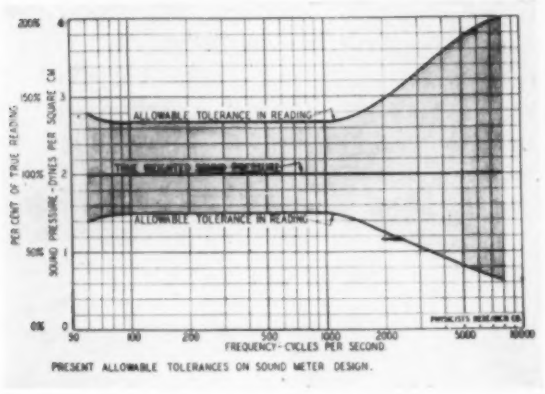

ALLOWABLE TOLERANCE IN READING
ALLOWABLE TOLERANCE IN READING
FREQUENCY-CYCLES PER SECOND
PRESENT ALLOWABLE TOLERANCES ON SOUND METER DESIGN.

FIG. 2. Present American Standards Association design tolerances for sound-level meters plotted to a pressure scale.

position is absolutely right. To date, there is no known instrument which weights and adds the components of a sound in the same way that the ear does. The differences are by no means small, and on practical sounds the discrepancy between meter measurements on equally loud sounds may be 10 db or more. As a practical matter, one must start with the fact that loudness cannot be measured on a meter. The only known methods for determining the loudness of a sound are to make ear comparisons with a group of observers, or to compute the loudness by means of very complex formulas which require extensive data concerning the acoustical spectrum of the sound.[1]

Sound Pressure versus Ear Impressions

Some people have argued that since the loudness of a sound cannot be measured on a sound level meter, such measurements were not worth taking. The author does not subscribe to this theory. Sound pressure measurements are of proven value. It seems axiomatic that the information which the ear obtains from a sound must reside in the magnitude and character of the sound pressure existing at the ears. These characteristics of the sound are physical, and are subject to physical measurement. Hence, physical measurements of sound pressure allow us to measure the character of the sound as heard by ear.

Early attempts to measure sound employed oscillograph records of sound pressure (recorded with less rather than more fidelity). The theory was that all of the information was there. If so, most of it is still there, because little has been obtained from such records.

Later, simplified records were obtained by the use of a thermoelement, or other rectifying unit. Then came harmonic analysis. All of these measurements are useful, but in many practical cases they did not yield usable information.

Use of Sound Pressure Measurements in Industrial Quieting

More recently, a different approach to sound measurements has been applied with great success. Instead of starting out as a physical problem to measure what was there, the more power-

FIG. 3. Residential substation showing power transformers

ful method is to determine what characteristics of the sound account for the observed effects. This "other end to" approach is a great time saver. Ordinarily one is interested in measuring the difference between two or more sounds. Occasionally a total sound-level measurement with

[1] Harvey Fletcher and W. A. Munson, "Loudness, Its Definition, Measurement and Calculation," J. Acous. Soc. Am. **5**, 82 (1933). Harvey Fletcher, "Loudness, Masking and their Relation to the Hearing Process and the Problem of Noise Measurement," J. Acous. Soc. Am. **9**, 275 (1938).

suitable frequency weighting serves as a suitable measurement. More often, one wishes to know the frequencies and magnitudes of the recurrent components (notes) of the sound. These can be measured with a suitable analyzer. On most of the common types of noise it is found that only a small fraction of the sound is recurrent. Hence, the methods of Fourier do not apply. This is often hard for physicists to believe, and they try to use harmonic analysis anyway, but of course it does not work and leads to absurdities.

The amount of unpitched or nonrecurrent component of any sound can be determined by measuring the total sound pressure, and then subtracting the combined sum of the recurrent components. On most machinery noises, the unpitched sound determines both the level and the quality, and the notes are comparatively unimportant. This fact has to be experienced to be appreciated.

Often it is desirable to measure the frequency distribution of the nonrecurrent part of a sound. This can be accomplished by suitable networks and filters.

Very often the important characteristics of a sound have to do with time variations in the magnitude or frequency, or both, of one or more of the components. Such time variations may occur in fractions of a millisecond, or over seconds or minutes. The ear is particularly sensitive to such changes, and it is often necessary to measure them. Suitable oscillographic equipment is often useful.

It is an axiom in our laboratory, that every noise job requires a different modification of equipment in order to measure satisfactorily the important characteristics of that job. By the same token, we have always been able to measure the physical characteristics of the sound pressure which controlled the observed effects. We take no stock in the statement, "I cannot measure any difference, but I can hear it with my ear." We have always been able to concoct methods for separating out for measurement the things which could be heard by ear.

When sound measurements are used in a program of machinery quieting, their principal value is in determining the mechanism by which the noise is produced, and the factors which control it. An experimenter with proper background can modify sound measuring equipment to measure fluctuations in pressures, flows, timing, displacements and vibrations of various parts inside and outside the machine under study. A correlation of such data with the sounds actually produced, furnishes an extremely powerful method for handling practical problems which are hopeless with ordinary means.

Noise versus Sales Resistance

After all, there is not much point in measuring sound unless something is done with the measurements or with the sound source. Most of us spend our lives immersed in a needless sea of noise. There is ample evidence that these noise levels are detrimental to our comfort, health and effectiveness. A major part of this noise comes from machinery of some sort, and this machinery must be quieted before much relief is obtained. In many cases, it costs no more to build quiet machinery than it does noisy, and sometimes the quieter machinery costs less. Quietness does, however, require attention in design, and manufacturers are not inclined to go to much trouble for noise reduction until noise interferes with sales. Just as soon as consumers in general really make a point of having quiet machinery, real efforts will be made, and it seems certain that much can be accomplished.

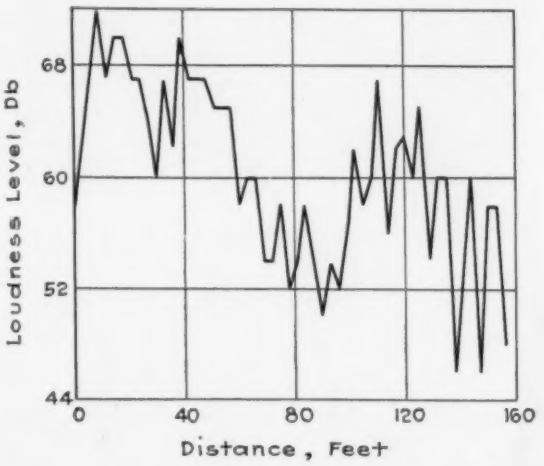


FIG. 4. Example of wave pattern: "inverse-square law" plotted against distance from fence shown in Fig. 3. Note that reading at 150 ft. is identical with that at zero feet.

Loudness versus Disagreeability

With measurements such as those outlined above, it is possible to compute the loudness of a sound by published formulas. Many people assume that the loudness is what one wants to know about a sound, but experience indicates that such is seldom the case. The two factors which have more to do with acceptability of a sound than any others appear to be: (1) Does the auditor think the sound is justified? (2) Does the auditor think the sound represents the normal functioning of the device? These far outweigh any considerations of loudness, general preference to certain tones, or any of the factors so often suggested. If one thinks a sound is justified, he will be happy in the presence of very high levels. If he thinks the sound is unjustified, he will object strenuously to levels that he has to strain to hear. In fact the strain will irritate him more than

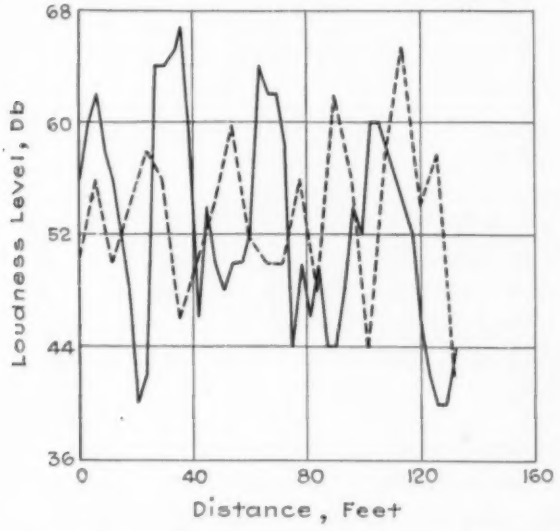


FIG. 5. Example of wave pattern: "equal-potential surfaces." At 160 ft. distance from source. The full line is with both transformers operating and the dotted line is with only one transformer operating. Note that at many points the level is greater when part of the source was removed.

the sound, and the ear can really go to low levels when strained!

Similarly, if the sound represents the normal functioning of a justified device, a high level may lull us to sleep. But let the sound change to an abnormal functioning, and see how disagreeable it becomes. These are real technical factors. Loudness can be determined if needed, but usually it is more profitable to find out what characteristics of the sound pressure are important in the case at hand.

Wave Patterns

Beyond any possible question, wave patterns are by far the greatest drawback to industrial sound measurements. Until actual measurements are made, it is impossible to visualize how much the sound pressure varies in magnitude and quality from point to point in any sound field. For this reason, observations made at any single point are practically worthless. Attempts have been made to avoid patterns by working close to the source, by acoustically treating the room, or by working outdoors. For the most part, these are futile. Any object more than a few inches in any dimension does not vibrate as a unit at acoustical frequencies, but vibrates in segments of differing amplitude and phase. Each of these segments generates its own train of sound waves, and interference between these waves gives wave patterns right up to the source. Most industrial noises have important components in the neighborhood of 100 cycles, and it is impractical to provide acoustical treatment to eliminate wave patterns in rooms at such frequencies. Windage noise makes it impossible to work outdoors except on loud sounds and on quiet days. Attempts to use a standard position for comparing different "identical" units usually come to grief because the units do not produce identical wave patterns.

Except in special instances, the practical way to overcome wave patterns is to average readings made at several points. This is costly, but must be done. Sometimes, swinging microphones, or moving reflector vanes are useful. If the sound is produced electrically, the frequency can be "warbled," but it is seldom feasible to warble machinery. Because of wave patterns, and the numerous other variables, it behooves one to plan acoustical work with a high ratio of thought to experiment. Otherwise, the number of readings reaches astronomical proportions, and life is too short.

On the Need for More Accurate Sound Pressure Meters

As long as meter specifications are given in unfamiliar decibels, it does not sound so bad to al-

low a tolerance of plus or minus 3 db when measuring a 90-db sound. In fact, it was only a few years ago that one of our prominent instrument manufacturers published a statement to the effect that there was little point in worrying about 5-db discrepancies in the meter, because the sound probably varied more than that anyway.

The absurdity of such tolerances is readily apparent when they are expressed in ordinary pressure units. Fig. 2 shows the present A.S.A. tolerances on sound level meters. When it is remembered that sound pressures are alternating quantities, and add root mean square unless the frequencies are identical, the actual effects are even greater than indicated.

One ordinarily uses sound measurements to determine the difference between two sounds, usually a "before" and "after" measurement of some sort. Suppose for example, that our noise is caused by two equal sources, and that our experiment has eliminated one of them entirely. Certainly, we have made great progress in the solution of our problem. Suppose, for example, our original components are of different frequency and 2 dynes per sq. cm each:

$$P_1 = 2 \text{ dynes per sq. cm.}$$
$$P_2 = 2 \text{ dynes per sq. cm.}$$

Original total "before"

$$(2^2+2^2)^{\frac{1}{2}} = 2.38 \text{ dynes per sq. cm.}$$

Now, if P_2 is removed, the sound pressure is 2.0, or a reduction of 30 percent. But the allowable tolerance of a present-day sound level meter is 100 percent of the smaller value. Hence, the complete removal of *half* the sound produces a change much less than the range of tolerance on the reading.

As in other physical measurements, an uncertainty of measurement of more than a few percent makes the method impractical for measuring changes.

At the present stage of the art, it is feasible to repeat measurements to a few percent in sound pressure with a single instrument (although many on the market will not do so). Different meters from the same source usually disagree by far more than this amount. Further, the readings of two meters cannot be correlated with a correction factor, because in general their frequency responses are different. For some sounds one will read high, while on other sounds, the other meter will read high, and there is no way to correct.

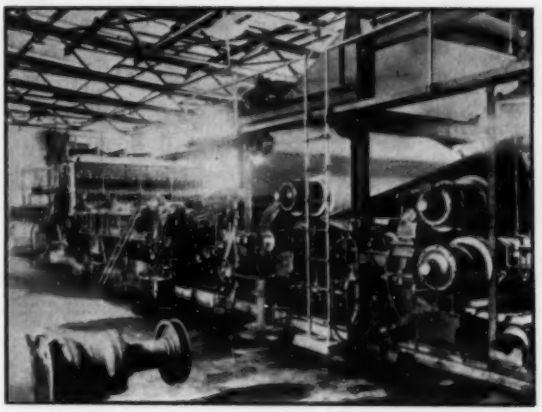


FIG. 6. Paper making machine. The suction rolls of this machine were quieted 20 to 30 db with the aid of sound measurements to determine the mechanism by which the noise was produced and the factors which affected it. The quieting devices did not affect the operation of the machine, and cost less than the parts which they replaced.

The National Bureau of Standards is now making absolute microphone calibrations for industry, and it is hoped that some of the discrepancies between meters will be eliminated.

Conclusion

The application of sound measurements in industry has been greatly retarded by a lack of suitable instruments, and a lack of appreciation of fundamentals such as those outlined above. Some years ago, the art suffered from overenthusiastic attempts at offhand solutions of difficult and complex problems. Following this, there was a period of suspicion. Meanwhile there have been outstanding successes where proper methods have been used, and an appreciation of the requirements and possibilities of the field is growing. It appears that the application of sound measurements to industrial problems is now definitely on the increase.

Resumés of Recent Research

Recent advances in experimental and theoretical physics are described in these columns in nontechnical language. It is intended that sufficient background be given to introduce the reader to the subject.

The Nature of Radio Fade-out

A radio fade-out is defined qualitatively as a sudden marked diminution or complete cessation of radio echoes from the ionosphere over the Earth's sunlit hemisphere. It is now well established that a fade-out coincides with a bright solar chromospheric eruption and on certain occasions with geomagnetic and earth-current changes of specific character. Because this is the first instance of time-coincidence of solar and terrestrial effects, the phenomenon offers unusual opportunity in investigation of solar and terrestrial relations. Association of these solar and terrestrial effects is due to Dellinger,[1] who in 1935 showed them to be coincident.

Cause of radio fade-out has been investigated by Berkner and Wells,[2] and more recently by Berkner[3] who observed echoes over a wide range of frequencies by automatic methods. When radio waves of intermediate or high frequencies are propagated during daylight there is an upper and a lower limit on the range of wave frequencies at which echoes from the ionosphere can be observed. This is illustrated in Fig. 1 for conditions at vertical incidence. The upper limit is imposed by penetration of waves through the ionosphere, while the lower depends upon absorption of waves passing through the lower ionosphere. During a fade-out, the lower or absorption-limit is projected upward through the frequency-range of normal echo-return, and intersects the upper or penetration limit during more intense effects so that no echoes on any wave frequency can then be observed (see Fig. 2). Analysis of this effect localizes the region of intense absorption of the radio wave during fade-out to a level between 60 and 100 km above the earth. Absorption occurs here because of intense ionization produced by radiation from the chromospheric eruption.

Measurements have been made in the regions above 180 km during partial fade-out, when they are not entirely obscured. Such observations show no indication of disturbance in these regions during a fade-out, indicating that the solar radiation causing fade-out passes through them without producing noticeable change. A small increase of ion-density is observed at the 110-km level. Thus solar radiation producing a fade-out is only absorbed appreciably at levels below about 100 km.

The geomagnetic effect accompanying a fade-out is shown by McNish[4] to be an augmentation

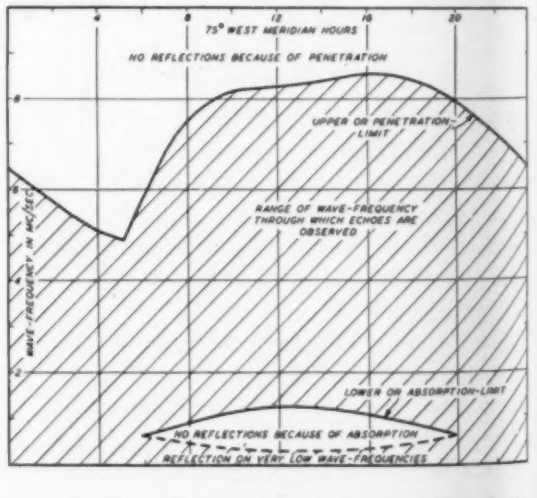


FIG. 1. Wave frequency boundaries giving reflection-limitation during normal conditions; drawn for six control days near day of fade-out of July 31, 1937, Kensington, Maryland (DTM 5201).

[1] J. H. Dellinger, Science **82**, 351 (1935); Phys. Rev. **48**, 705 (1935); Terr. Mag. **42**, 49–53 (1937).
[2] L. V. Berkner and H. W. Wells, Terr. Mag. **42**, 183–194 (1937); Terr. Mag. **42**, 301–309 (1937).
[3] L. V. Berkner, Phys. Rev. **55**, 536–544 (1939).
[4] A. G. McNish, Nature **139**, 244 (1937); Terr. Mag. **42**, 109–122 (1937); Phys. Rev. **52**, 155–160 (1937).

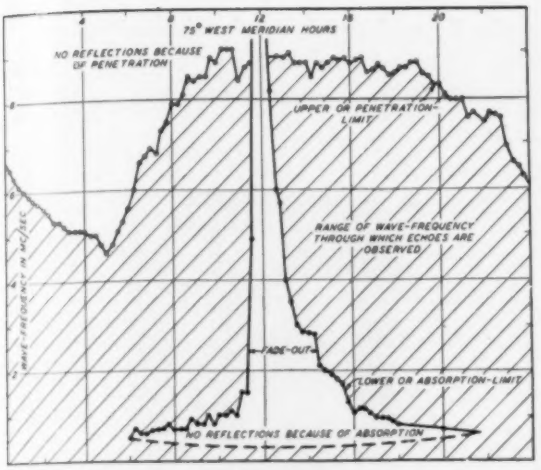

75° WEST MERIDIAN HOURS
NO REFLECTIONS BECAUSE OF PENETRATION
UPPER OR PENETRATION-LIMIT
RANGE OF WAVE-FREQUENCY THROUGH WHICH ECHOES ARE OBSERVED
FADE-OUT
LOWER OR ABSORPTION-LIMIT
NO REFLECTIONS BECAUSE OF ABSORPTION

FIG. 2. Wave frequency boundaries giving reflection-limitation during fade-out; illustrating upward projection of lower or absorption-limit for fade-out of July 31, 1937, Kensington, Maryland (DTM 5202).

of geomagnetic diurnal variation. Because increase of ionization necessary to produce this effect appears only in the atmospheric region between about 60 and 100 km, as localized from the fade-out studies, it appears probable that geomagnetic diurnal-variation is produced by a current-flow between these limits of height in accordance with the Stewart-Schuster atmospheric dynamo-theory.

Reversed Speech Recording sound and then reproducing it with the rotation of the record reversed is not only a means of amusement, but serves to throw some light on our habits of speech, particularly if one tries reversing the order of the original pronunciation, so that the reproduced sound may be actual words. A recent paper by E. W. Kellogg[1] illustrates this point.

When one talks, he seldom makes the sounds he thinks he does. Many of the sounds which ought to be present are slighted, or inexact substitutions are used. This is particularly true of short vowels. Just as a mother can recognize her baby in spite of a very dirty face, so we recognize familiar words provided a reasonable fraction of their characteristics is preserved. In the next place, speech is understood by recognizing whole words and not by a laborious synthesis of the word from a succession of sounds. A speed of speech which is entirely satisfactory when words can be recognized becomes many times too rapid when trying to identify the individual sounds in a meaningless sequence. One thus finds that he gets farther in learning about the reversibility of speech if he attempts to say the words backward and then listens to the result.

One of the first things which is forced on one's attention is that a number of the vowels which he represents by a single letter and usually thinks of as a single sound are very distinctly a sequence of two sounds. For example, "ī" must be reversed to "ē–ā," "j" as usually pronounced consists in "d" followed by a soft "j," and "ch" must be reversed to "sh–t." Overlooking any of these details produces an unexpectedly striking fault in the reversed sound.

In general final consonants are much less conspicuous than initial consonants. The final consonants *p*, *t*, *k* are often lost, and their absence does not seriously impair one's understanding of the spoken word.

As an illustration of talking backwards, the author gives a few lines spelled phonetically to the best of his ability. If you wish to learn what the corresponding English is, start from the bottom right-hand end and pronounce the sounds as you come to them from right to left.

"Tiónoot rawf tsujd négyĕ dliósht yĕ eem kiém dnă. (1)

"Tiólf yŏth nĭ miŏt oh drawkáb nrŭt drawkáb." (2) (3)

(1) Soft "j," like French.
(2) Voiced "th."
(3) Pronounced as word "draw" but with the "w" emphasized.

Diffusion in Metals Using Radioactive Indicators The processes of atomic interchange which account for diffusion proceed in a pure chemical element as well as in phases containing several elements. In order to study the diffusion of a chemical element into itself—self-diffusion—it is necessary that some of the atoms of the element be distinguishable from the others. Advances in

[1] E. W. Kellogg, J. Acous. Soc. Am. 10, 324 (1939).

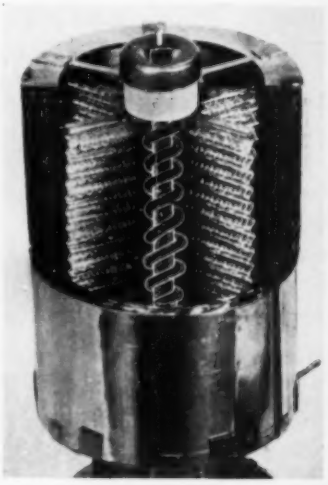
the techniques of nuclear physics have now made available a number of radioactive isotopes, and these are in many cases quite suitable to act as "tagged" atoms in self-diffusion studies. The results of a study of the self-diffusion of copper using a radioactive isotope have recently been reported by Steigman, Shockley and Nix.[1] In this work the diffusion of a thin, electroplated deposit of radioactive copper into a block of inactive copper was studied by observing the decrease in apparent activity: as the radioactive copper penetrates the inactive, it becomes shielded and the intensity of its emerging radioactive radiation diminishes. From the rate of diminution in intensity, the rate of diffusion is found. Von Hevesy was the first to use this indirect technique for determining diffusion rates. It short-cuts the usual methods of cutting away thin sections of the diffusion specimen and analyzing them chemically; and although the more laborious technique is necessary in some instances, it may be possible to supplement it advantageously with newer and quicker method.

[1] Steigman, Shockley and Nix, Phys. Rev. **56**, 13 (1939).

A New Long-Life Cathode for Thyratrons

In the early days of vacuum tubes a life of 1000 hours was considered satisfactory. Gas-filled rectifiers and Thyratrons found new industrial applications, which demanded a life of 10,000 hours. This has been attained with oxide-coated cathodes of proper design. There now are appearing on the horizon further applications in the electric power field, which require Thyratrons with an operating life of 100,000 hours or more, at high electron-emitting efficiency. Promise that such tubes may be forthcoming is contained in a paper by Dr. A. W. Hull of the Research Laboratory of the General Electric Company. This paper describes a new type of thermionic cathode, called the *dispenser cathode.**

The dispenser cathode is similar to the heat-shielded oxide-coated cathode, except in the method of applying the coating; the coating material is dispensed to the cathode members gradually and continuously throughout life, instead of being applied all at once. In a typical structure (Fig. 1) the electron-emitting members



are radial vanes of clean molybdenum, which are both heated and coated by a tubular filament of molybdenum gauze, filled with activating material. The life of these cathodes depends on the amount of activating material in the tubular filament, which may be ample for many years of operation. Test cathodes show undiminished electron emission after three years of operation.

* A. W. Hull, Phys. Rev. **56**, 86 (1939).

New Studies of the Oxide-Coated Filament

A quantitative correlation between the thermionic emission of an oxide-coated filament and its content of free alkaline-earth metal is reported by C. H. Prescott, Jr., and James Morrison in recent communications from the Bell Telephone Laboratories.[1–3] Although oxide-coated filaments were used in some of the earliest practical vacuum tubes, knowledge of their structure and the relations between physical properties and chemical constitution is largely

[1] C. H. Prescott, Jr., and J. Morrison, "The Oxide-Coated Filament. The Relation Between Thermionic Emission and the Content of Free Alkaline-Earth Metal," J. Am. Chem. Soc. **60**, 3047 (1938).

[2] C. H. Prescott, Jr., and J. Morrison, "The True Temperature Scale of an Oxide-Coated Filament," Rev. Sci. Inst. **10**, 36 (1939).

[3] C. H. Prescott, Jr., and J. Morrison, "Apparatus for Micro Gas Analysis," J. Ind. & Eng. Chem.

uncertain, owing to the minute quantities of active metal associated with thermionic emission, and the complexity of the chemical system in which they are situated. Accurate studies have required stringent control of conditions and the development of highly sensitive methods of analysis.

A filament was used having a coating of barium oxide, strontium oxide and finely divided nickel on a platinum-rhodium core. It was activated by glowing in methane according to the reaction.

$$CH_4+BaO=Ba+CO+2H_2.$$

A high level of activity was found from 15 to 60 micrograms per square centimeter of free alkaline-earth metal, computed as equivalent barium. A slight maximum appears at 30 micrograms per square centimeter, where the thermionic current at 1050°K is 600 milliamperes per square centimeter. The electron work function is 1.37 volts.

A necessary part of this work was the development of a method to determine the true temperature of the filaments. Using the relation

$$\text{Reflectivity}+\text{emissive power}=1$$

the corrections to temperatures measured directly with the optical pyrometer are estimated from measurements of the diffuse reflectivity. The experimental tube, but minus the anode plates, was placed in the axis of a cylinder lined with white cotton velvet, illuminated by two lamps coaxial with the filament. The reflectivity is computed from measurements of the apparent temperatures of the filament and the back wall of the cylinder. For these filaments the emissive power averaged 68 percent independent of temperature and independent of the degree of activation, i.e., the content of free alkaline-earth metal.

The free alkaline-earth metal is estimated by oxidation with carbon dioxide and determination of carbon monoxide in the gaseous reaction products. To attack this problem, gas analysis apparatus and methods have been developed for which the least detectable quantity of carbon monoxide is 0.025 cubic millimeter at normal temperature and pressure, an amount which would occupy one square centimeter of a monomolecular film.

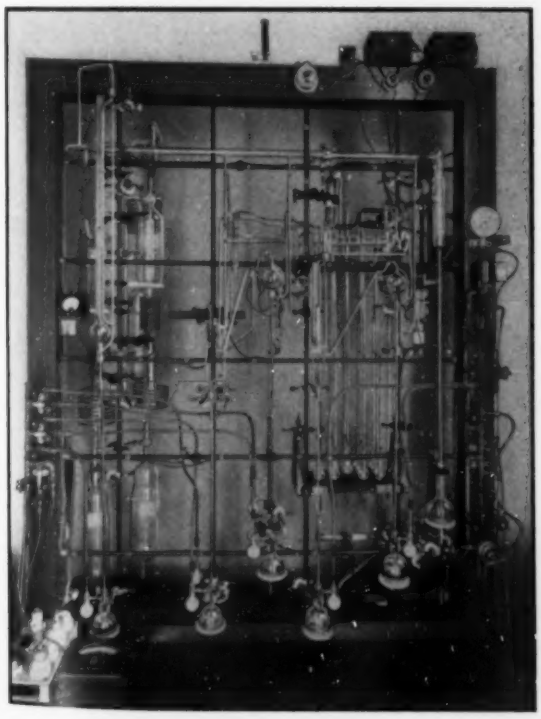


FIG. 1. Gas analysis apparatus used in the estimation of free alkaline-earth metal. (Courtesy of Industrial and Engineering Chemistry.)

Recent Developments in Pressure-Insulated Electrostatic Generators

Since the development by Van de Graaff[1] of the belt-type electrostatic generator at Princeton in 1931, such generators have become of increasing importance as the high voltage source in nuclear research, in the production of high voltage x-rays for cancer treatment, and in other applications. The requirement for voltages of the order of several million in compact apparatus has led within the last few years to the use of compressed gases rather than air at atmospheric pressure as the insulating medium. It is one of the striking advantages of this type of voltage source that without any substantial change in the apparatus itself an increase in the dielectric strength of the medium in which the generator is housed should result in a corresponding increase in its voltage rating. The large electrostatic generators[2] which

[1] R. J. Van de Graaff, Phys. Rev. **38**, 1919 (1931).
[2] W. H. Wells, J. App. Phys. **9**, 677–689 (1938).

have recently been constructed by the Westinghouse Company and by the Carnegie Institution following the initial pressure-insulation work at Princeton University[3] and by Herb[4] at Wisconsin are applications of this idea.

At the Massachusetts Institute of Technology, where the large air-insulated generator originally constructed at Round Hill has been installed for nuclear research, there has also been a program directed at the use of electrostatic generators for the production of high voltage x-rays. The first x-ray generator developed by Technology[5] has now been in use for cancer therapy at the Huntington Memorial Hospital in Boston for over two years, during which time treatments have been given to about 700 patients with some promising medical results.[6] This machine operates at 1000 kilovolts and is housed in a room 25′×23′×20½′ high.

Recently a development has been completed at Technology aimed at the more effective application of compressed gases to electrostatic generators. The new x-ray generator,[7] which is housed in a pressure chamber 34 in. in inside diameter and 100 in. high, has been used for physical measurements at voltages up to 1500 kilovolts. This machine is shown alongside with the pressure chamber removed. It is intended for therapy use at voltages up to 1250 kilovolts and will later be installed at the Massachusetts General Hospital for this purpose. In the course of this development several problems fundamental to the operation of electrostatic generators in compressed gases became recognized and were met, with the result that for the first time in such generators both the voltage and the current rating increase nearly proportionally with the pressure of the gas employed. It is to be expected that because of the simplicity and reliability inherent in electrostatic high voltage sources, as well as because of the value of extremely constant voltage in many physical experiments, compact compressed-gas-insulated electrostatic generators will play an increasingly important role in physics, engineering, and medicine.

[3] H. A. Barton, D. W. Mueller and L. O. Van Atta, Phys. Rev. **42**, 901 (1932).
[4] R. G. Herb, D. B. Parkinson and D. W. Kerst, Phys. Rev. **51**, 75 (1937).
[5] J. G. Trump and R. J. Van de Graaff, J. App. Phys. **8**, 602 (1937).
[6] R. Dresser and J. C. Rude, J. Am. Med. Assoc. **111**, 1834 (1938).
[7] J. G. Trump and R. J. Van de Graaff, Phys. Rev. **55**, 676 (1939).

Applied Physics in the Bureau of Home Economics

BY MARGARET B. HAYS

Associate Textile Physicist, Textiles and Clothing Division, Bureau of Home Economics, United States Department of Agriculture

Introduction

THE United States Bureau of Home Economics, authorized to study the utilization of farm products in the home, has four subject matter divisions: foods and nutrition, economics, textiles and clothing, and housing and household equipment. With the exception of the economics division which is concerned chiefly with studies of family income and expenditures, all the others use physical methods for evaluating quality in consumer goods. The problems cover a wide range of subjects, and include studies on the leavening power of egg whites in making cakes, on the serviceability of wool when made into blankets, and on the efficiency of various cooking devices. In fact physics is such an integral part of the home economics research in these laboratories, that it is taken completely for granted.

Foods and Nutrition

The place of the minerals such as calcium, phosphorous and iron in human nutrition is well defined. Research work now deals with trace elements to see if they too play an important role even though present in minute quantities. Vanadium one of these trace elements is being investigated and its distribution in animal tissue is measured quantitatively with a prism spectrograph (Fig. 1). Another optical instrument is the visual adaptometer (Fig. 2). This measures the reaction of the human eye to light of different intensity. A prolonged deficiency of vitamin A in the diet leads to nutritional night blindness when the individual may need 10,000 times as much light as normal to see in dim light. In operating the visual adaptometer, the visual purple of the retina is first bleached by exposure to a bright light and is then allowed to rebuild during a period of dark adaptation. Measurements of the light thresholds at intervals during the dark adaptation period will show whether or not a state of night blindness exists. By periodically testing people on experimental diets with the adaptometer, it is possible to establish the daily vitamin A diet needs of a normal adult.

FIG. 1. This spectrograph is used in the Bureau of Home Economics for the analysis of trace elements in foods.

FIG. 2. The light sensitivity of the human eye is studied by means of a visual adaptometer.

In addition to information on the body requirements of various nutrients, there is a further need of objective ways of measuring food quality. Workers in food preparation realize that the proof of the pudding is still in the eating but they want a method of judging food independent of the personal bias introduced by taste. Physical

FIG. 3. The thermometer is inserted in the cut of beef before roasting.

instruments are being developed to eliminate, or to check the personal element in evaluating food products.

One instance is the nation-wide study which has been in progress for the past ten years on the palatability of meats as affected by breed, feed, sex, and method of handling the meat after slaughtering. Certain cuts of beef, lamb, and pork[1] are cooked by standardized methods in controlled temperature ovens. Also a special meat thermometer (Fig. 3) inserted directly into the roast tells exactly when the center is rare, medium, or well done and enables each cut to be cooked to a specified state. While a board of judges tastes and scores the cooked meat, tenderness is measured in terms of shearing force by a special device (Fig. 4). The results of these mechanical shears often correlate quite closely with those of the human judges.

As a by-product of these studies on the inherent qualities of the meat itself, more scientific methods of cooking meat in the home are being developed.

Another project yielding practical as well as scientific data is the one of the lifting power of thick and thin egg whites when the foams are used in angel and sponge cakes.[5] Tests are made on the cake to measure volume, tensile strength (toughness) and compressibility (softness).

The breaking strength of cookies and pastries made with various fats is measured in a shortometer (Fig. 5). The penetrometer (Fig. 6)

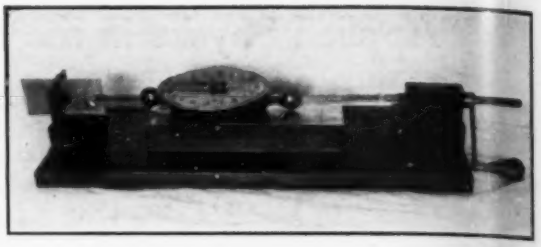


FIG. 4. This machine measures the force required to shear a sample of meat. (Courtesy Bureau of Animal Industry.)

FIG. 5. Breaking strength of pastry is determined in studies on the shortening property of fats.

FIG. 6. The consistency of a fat or the strength of a jelly is measured with the penetrometer.

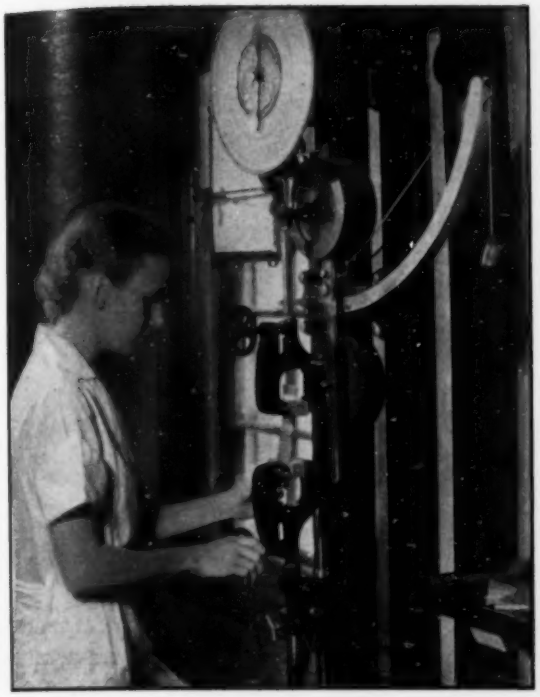


FIG. 7. A breaking strength determination of an upholstery fabric is in progress in the air-conditioned textile laboratory.

for testing the consistency of fats can also be used to measure the strength of gels formed by thickening agents such as starch, gelatin and pectin.

Still another instrument is being developed which will measure the force required to cut through cooked potatoes so that their texture can be estimated objectively. The relationship between the specific gravity of potatoes and their mealiness when cooked is being investigated also.

Textiles and Clothing

Strength is one of the most important properties of fabrics and is determined regularly as a part of all studies in the division of textiles and clothing under the constant conditions of 70°F and 65 percent relative humidity. However, the cross-sectional area of a fabric is difficult to determine, so an empirical breaking test, instead of tensile strength, is made. Fig. 7 shows a "strip" test in progress on a machine equipped with an automatic recorder for elongation at break which applies the load at a constant speed. Another test designed for knit fabrics is the bursting strength, in which the fabric is pulled down over a steel ball by an attachment applied to this same machine. In both cases, the number of pounds required to rupture the material is recorded on the dial.

Since certain starches increase the strength of a sized fabric, the breaking strength and elongation of starch films have been investigated.[3] The stiffness produced in a fabric is evaluated by a stiffness tester (Fig. 8).

The textiles studied range in thickness from chiffons to blankets. However, thickness of textiles is simply a matter of definition, because the value obtained is dependent on the pressure applied to the surface of the specimen. The

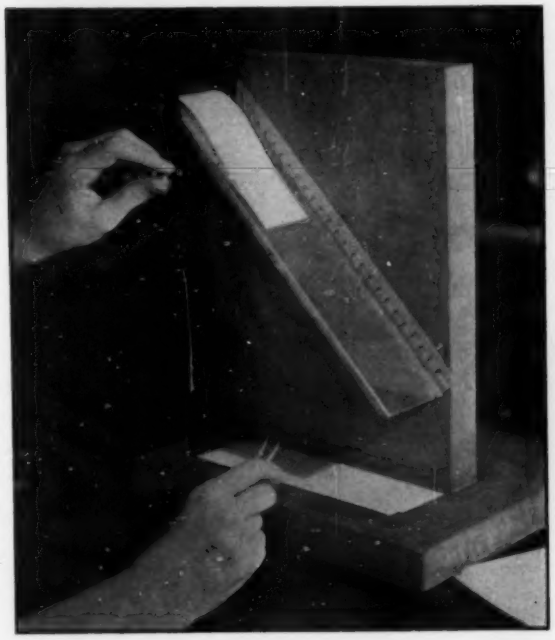


FIG. 8. The stiffness tester measures the deformation at an angle of 45° of a horizontally supported strip of material bending under its own weight.

compressometer (Fig. 9) measures thickness as the pressure is increased from 0.1 to 2.0 pounds per square inch and as the pressure is decreased. Standard thickness is defined as the thickness when the pressure is increased to one pound per square inch. This same instrument can be used to evaluate the compressibility and compressional resilience.

The relation of fiber property to the finished fabric is determined by manufacturing fibers of

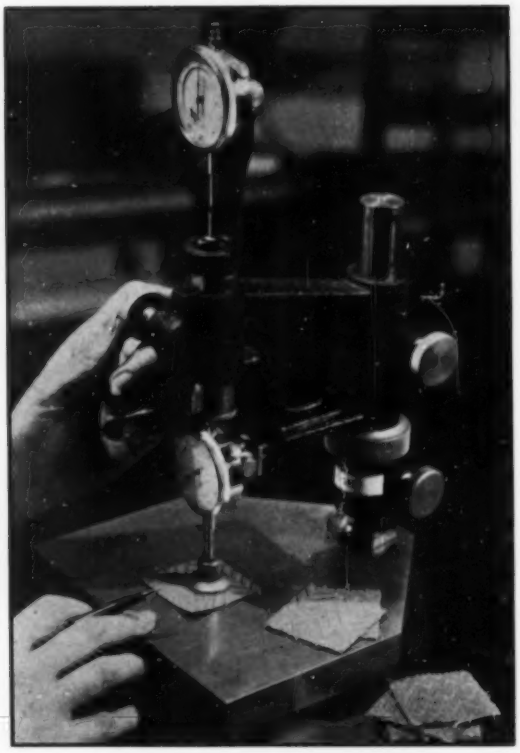


FIG. 9. The compressometer measures the thickness of upholstery materials at different pressures.

known history into comparable fabric constructions—for example, wool into blankets. The physical properties of the fiber are studied through the cross-sectional variability and the scale structure[4] when new and after the blanket has been used. Warmth of a fabric is evaluated

FIG. 10. Air permeability is determined for blankets at a pressure difference of one pound per square foot.

FIG. 11. The fabric samples are abraded by holding them against an oscillating drum.

by measuring the amount of heat transmitted when one surface is kept at body temperature and the other is exposed to air in the conditioned laboratory. Another factor in warmth is the amount of air that will pass through the fabric at a known pressure difference. It is measured with an air permeability apparatus (Fig. 10).

Resistance to abrasion is another important test for textiles, but one that is not standardized. There are many such machines that are now used, but no one is suitable for all fabrics. The one shown in Fig. 11 controls the pressure and tension, but does not flex the fabric during the test. The abradent used in this instrument is a stainless iron gauze, a surface which changes slowly with use, and the effect of the heat factor is unknown. Change in breaking strength in the opposite direction to that of abrading gives a relative measure of resistance to abrasion of the different materials in a study of upholstery fabrics,[6] while change in thickness is used to evaluate cotton corduroys. Another abrasion tester simulates conditions of use for the toe and heel reinforcements of hosiery.

For testing the color permanence of materials to washing, a launderometer controls the temperature and mechanical agitation during washing. The fadeometer, using a carbon arc and a corex-D globe, simulates sunlight (Fig. 12) for studying the fastness of dyed fabrics to light.

The surface reflectance of fabrics is determined by spectrophotometric measurements at 405, 436, 495, 546, 651, and 703 $\mu\mu$. The intensity of light reflected by the test sample is measured with reference to a standard magnesium oxide surface (Fig. 13). For a study on the effect of

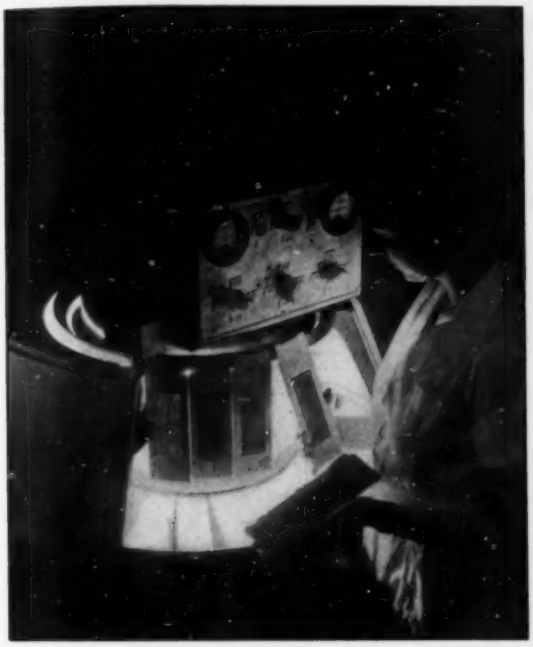


FIG. 12. Dyed fabrics are exposed to carbon arc light in the fadeometer.

variations in pressure and in length of time upon exposure during ironing on cotton fabrics, the wave-lengths 405 and 436 $\mu\mu$ were effective in detecting slight surface changes due to ironing.[2]

Housing and Household Equipment

Investigations of various types of household equipment involve the application of the laws of mechanics, heat and electricity.

In testing the efficiency of surface burners on cooking stoves using different fuels, in the housing and household equipment division, measurements are made of the energy delivered by the fuel and that absorbed by water in being raised to the boiling point. In the case of gas stoves, standard pressure is maintained by the use of gas pressure regulators and measured by manometers. The efficiency of refrigerators is measured in terms of the internal temperature and the fuel consumption.

For studies on either ovens or refrigerators, the temperatures maintained in different locations are measured by the use of thermocouples and potentiometers. Fig. 14 shows an electric roaster being tested by this means for the distribution of heat at a given setting of the thermostat. The rate of heat loss for any oven is found by the use of a temperature-time curve plotted from the data as measured by the thermocouples and potentiometer.

All studies of electrical household equipment are made using standard voltage which is secured and maintained by the use of a transformer and induction regulator. Measurements of current,

FIG. 13. The partial spectrophotometer measures the light reflected from the fabric surface.

FIG. 14. Biscuits serve as a test load in studying the heat distribution of an electric oven.

voltage and electrical energy always are included in these tests.

These applications of physics cited are merely ones in use on research projects now in progress or recently completed in this Bureau. The field of home economics also includes lighting, heating, air conditioning, and insulation of buildings for heat and sound. These require the cooperation of engineers and home economists, as well as physicists.

References

1. "Shrinkage and heat penetration during the roasting of lamb and mutton as influenced by carcass grade, ripening period, and cooking method," L. M. Alexander and N. G. Clark, U. S. Dept. Agr. Tech. Bull. 440, 26 pp., illus. (1934).
2. "Cotton fabrics as affected by variations in pressure and in length of exposure during ironing," K. M. Downey and R. E. Elmquist, U. S. Dept. Agr. Tech. Bull. 517, 32 pp., illus. (1936).
3. "Breaking strength, elongation and folding endurance of films of starches and gelatin used in textile sizing," M. S. Furry, U. S. Dept. Agr. Tech. Bull. 674, 36 pp., illus. (1939).
4. "A serviceability test on blankets made from four blends of wool," M. B. Hays, R. E. Elmquist and J. I. Hardy, U. S. Dept. Agr. Tech. Bull. 572, 24 pp., illus. (1937).
5. "Some methods and apparatus used in measuring the quality of eggs for cake making," F. B. King, H. P. Morris and E. F. Whiteman, Cereal Chem. 13 (1): [37]–49 (1936).
6. "Proposed minimum requirements of 3 types of upholstery fabrics based on the analysis of 62 materials," B. V. Morrison, M. B. Hays, U. S. Dept. Agr. Cir. 483 (1938).

The Nature of the Metallic State*

WILLIAM SHOCKLEY

Bell Telephone Laboratories, New York, New York

THE nature of the interatomic forces in metals is obviously of great significance in metallurgy. These forces must, in the last analysis, account for the characteristic and important properties of metals and alloys. Attempts to understand these forces in terms of the customary chemical concepts of atomic valence have not proved very fruitful, and the reason for this lack of success can now be satisfactorily explained in the light of recent advances in the theory of metals. During the last six years, there has been developed the first quantitative theory of the metallic state, and it is now possible to calculate, starting from first principles, many of the fundamental properties of the simpler metals, such as specific heat, thermal expansion, heat of sublimation, compressibility, and electrical conductivity. The quantitative success of the theory is due to the development of mathematical methods especially suited for the problems of metals. However, along with this mathematical apparatus, there have grown up certain new fashions of thinking and certain physical pictures which embody, in a simpler form, the same aspects of the theory that give quantitative results when treated mathematically. It is the purpose of this article to describe in non-specialized language the nature of these pictures and show how they explain the properties of the alkali metals, particular emphasis being placed on sodium, for which both the theoretical and experimental work are well advanced.

The metallurgist, I am sure, cannot entirely suppress a feeling of annoyance when he finds that the physics of metals is about to be discussed for the case of the alkali metals. He knows that these elements in the metallic state are of little technological importance and are extremely inconvenient to deal with even under laboratory conditions. It seems to him that the physicist is influenced by some sort of obtuse perverseness in choosing such valueless cases for investigation. The physicist similarly is somewhat irked to find that metallurgy concerns itself largely with such complicated cases as the iron-carbon system, whose complexities he cannot at present hope to unravel. It is probably simpler for the physicist to understand the metallurgist's activities than vice versa: for it is clear that in most cases the metallurgist is not entirely a free agent and must to a large extent devote himself to those activities which will produce practical results. The physicist, on the other hand, may choose his problems with considerable freedom but in his choice he sensibly selects those which he has a reasonable chance of solving. In each new field this means selecting the simplest problems, and one of my endeavors in this article will be to show why the alkali metals are, from the theoretical physicist's point of view, the simplest.

The tool which has been so successful in solving problems of an atomic nature is the wave equation of Schroedinger. This equation is used in calculations of the motions of electrons; and as its name implies, it describes these motions in terms of wave patterns. The simplest and most familiar wave patterns associated with electrons are those obtained in electron diffraction experiments. Such patterns obtained by Davisson and Germer gave the first evidence of electron waves and have since acquired an importance which rivals that of x-ray diffraction patterns in analyzing the structure of matter. At first Schroedinger's equation was applied to the simplest systems consisting of free atoms and ions and later to molecules. It was also applied to ionic crystals, which represent the simplest type of solid, being essentially just a stacking together of free ions. However, in the case of metals, there were difficulties which arose from the presence of the so-called free electrons.

* Based on a lecture given at the "Round Table Discussion of the Nature of Hardness" organized by the Physics of Metals Committee at the Metals Congress, Detroit, October 19, 1938.

The free electrons give metals the characteristic properties of high conductivity and metallic luster. These electrons are free in the sense that they can readily yield to the influence of an electric field and thus move through the crystal producing a current. By studying the behavior of this current for the case of the rapidly oscillating electric fields present in light waves, that is, by studying the optical properties of metals, it is possible to determine experimentally the

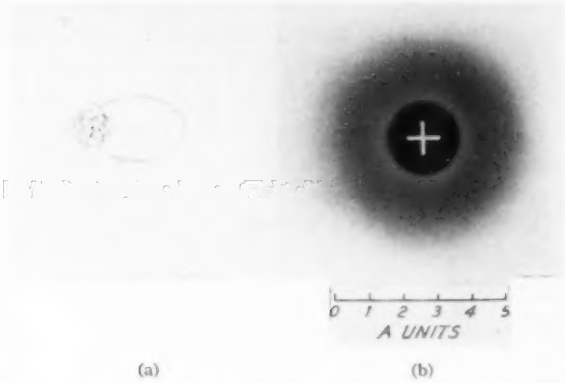


FIG. 1. Distribution of electrons in sodium atom. (a) Bohr theory, (b) wave-mechanical theory.

number of free electrons in a metal. For the alkali metals it is found that each atom furnishes one free electron. Although the properties of the free electrons were fairly well understood and their presence welcomed so far as electrical properties were concerned, they could not at first be fitted into a coherent scheme which would describe the other properties of a metal as well. It was not until 1933 that the work of Wigner and Seitz opened a new field of calculation in metals which has lead to the results to be described below.[1]

Where do the free electrons come from? To answer this for the case of metallic sodium let us consider a free sodium atom. Such an atom consists of a nucleus having a charge of plus eleven units surrounded by eleven electrons each having minus one unit of charge, so that the atom as a whole is electrically neutral. The nucleus is about 4000 times heavier than the eleven electrons and it may be regarded as a relatively immobile center about which the electrons move —much as the sun is the center of our planetary system. The eleven electrons are not equivalent; instead they can be broken into two groups: one valence electron, which is the one, as we shall see later, that becomes free in the metal, and ten core or ion electrons. This division is a direct consequence of certain well-established quantum-mechanical laws, the same laws, in fact, which explain the construction of the periodic table of the elements and in doing so furnish the most beautiful justification for the whole quantum-mechanical structure. However, limitations of space restrict the discussion of these laws to our particular case. For the sodium atom the valence electron moves in a wide orbit compared to the core electrons which are relatively compactly bound to the nucleus. It is a consequence of the laws of quantum mechanics that such phrases as "moves in an orbit" lose their precise meaning when applied to electrons. We must not think of the electrons as following definite paths about the atom as indicated in Fig. 1(a). Instead we must think of their position as being more vaguely specified as indicated in Fig. 1(b). In this figure the shading represents the average distribution of charge produced by the electrons; according to the quantum theory the motion of the electron is described in terms of wave functions, which do not state exactly what path the electron follows but instead tell only the probability of the electron being at a particular place. In Fig. 1(b) the outer diffuse ring of charge represents the wave function of the valence electron. In a three-dimensional drawing this ring would become a spherical shell. The inner dark region represents the highly concentrated charge distribution of the 10 core electrons. As a consequence of this arrangement it requires a comparatively small energy to remove the valence electron from the atom leaving behind the core electrons and the nucleus, which together constitute a sodium ion. Atomic energies are often expressed in electron volts: one electron volt is the energy required to move one electron through a potential difference of one volt. Accordingly, the energy to strip the valence electron from sodium, that is, the ionization energy of the sodium atom, is 5.12 electron volts.

The core electrons are much more tightly bound; in order to remove a core electron from the sodium ion 47 electron volts would be re-

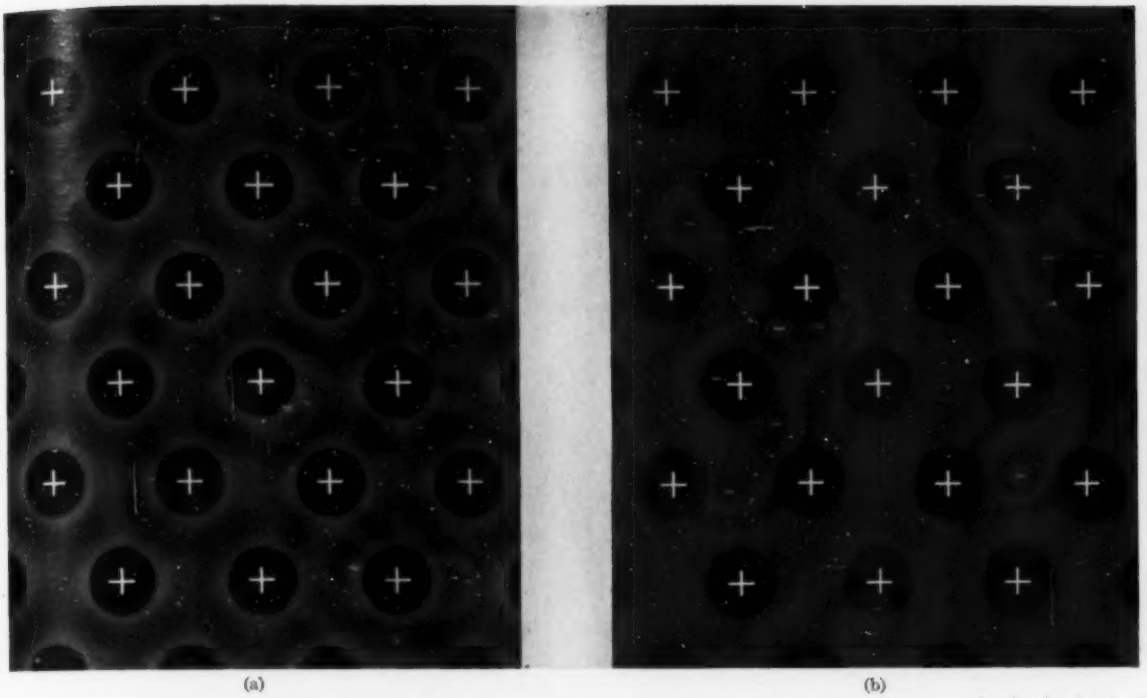


FIG. 2. Arrangement of sodium atoms in the 110 plane of the metal: left, the relative size of the atomic distributions; right, the theoretically predicted distribution of electrons.

quired. This energy is so high that, for our purpose, we may neglect the possibility of anything happening to the ion and regard it as an indivisible unit about which the valence electron moves.

Let us next consider the behavior of these valence electrons in the solid and see how they become free. Fig. 2(a) shows the arrangement of the atoms in the 110 plane of metallic sodium. We see that the atoms are so close together that the valence electron of a given atom approaches very close to other atoms—so close in fact that it can escape from its parent atom and go to a neighbor and thus become free to wander about the lattice. This description is, of course, highly pictorial: the correct way to treat the problem is to apply Schroedinger's equation to the valence electrons moving in the fields of the ions and to investigate the mathematical form of its solution. However, when the answer has been so obtained, it is found that in a number of its most important aspects, the solution corresponds to the following rather simple physical picture.

The valence electrons have become so free that they wander about the crystal and are no longer localized near their atoms. They give rise in this way to a rather uniform smear of negative electricity in which the positive ions are embedded, as is indicated in Fig. 2(b). This uniform smear should not, however, be thought of as a rigid solid but rather as gas for which the free electrons are the particles. We then think of the solid metal as a uniform gas of free electrons in which the ions float freely. This description seems at first sight to represent a structure far too flimsy and flexible to explain the rigidity of solid metals, but as we shall see it gives results in excellent agreement with experiment for the rigidity or shear moduli and also for the compressibility over a very wide range of pressures.

The Energy of the Alkali Metals

Let us consider how the energy of a given sample of material varies as we subject it to pressure, thus compressing the unit cell but not altering its shape. This energy is the kinetic energy of motion of the electrons in the electron gas plus the electrostatic interactions among these electrons and the ions; no thermal energy is included and the kinetic energy of the electrons

is a consequence not of thermal agitation but of the quantum mechanics of the electron gas. Calculations of the energy E for various values of the volume V have been carried out for sodium by J. Bardeen[2] and the results are shown in Fig. 3. In the figure we have used not the volume but instead the lattice constant or length of the edge of the unit cell. There is no exact analytical form for this curve, but to a satisfactory degree of approximation it is represented by the equation

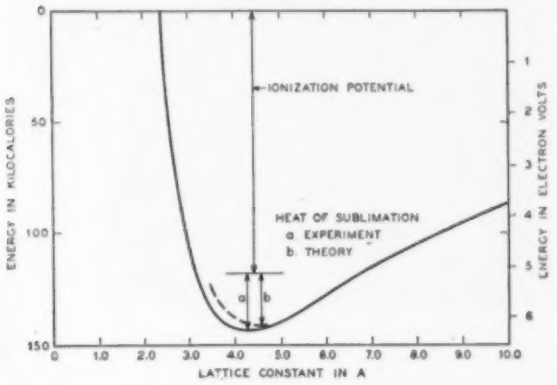


FIG. 3. Energy in kilogram calories per gram atom and electron volts per atom *versus* lattice constant, (a) theoretical curve, (b) three-term approximation to experimental curve as given by Table II.

$$E=A/V+B/V^{\frac{1}{3}}-C/V^{\frac{1}{3}}, \qquad (1)$$

where A, B and C are constants determined from atomic theory. At the absolute zero of temperature and at zero pressure, the stable state of the system is represented by the minimum of the curve. Of course, in any energy expression the state corresponding to zero energy can be arbitrarily chosen; for Eq. (1) it has been chosen as the state for which the electrons and ions have been completely separated. Thus we see that to transform the material from the state of lowest energy to a state in which ions and electrons are separated requires 6.13 ev per atom or 141 kilocalories per gram atom. This energy exceeds the energy required to sublimate the material into free neutral atoms by an amount equal to the energy of ionization of the atoms. Hence we find for the heat of sublimation or heat of formation the value $6.13-5.12=1.01$ electron volts per atom or 23 kilocalories per gram atom. This value was obtained entirely from theory and is in good agreement with the experimental value of 26 kilocalories per gram atom. The minimum of the theoretical curve should give the equilibrium configuration of the metal. For it we find a lattice constant of 4.53A (1A=1 angstrom $=10^{-8}$ cm) while the experimental value is 4.25A. From the curvature of the curve at its minimum one can determine the compressibility K. A comparison of theoretical and experimental values is given in Table I.

Let us next return to our picture of the metal and trace in it the origins of the three terms in Eq. (1). The B term is the simplest: it arises directly from a consideration of the electron gas. When the atoms are closely packed together as is suggested in Fig. 2, each electron moves in the field of a number of positive ions which attract it and negative electrons which repel it. Since there are equal numbers of ions and electrons, these forces balance each other to a first approximation and the electrons behave as if they were uncharged particles moving in empty space. Such a system of chargeless particles constitutes a gas obeying certain gas laws—not, however, the laws of an ordinary gas but instead certain laws well known in quantum mechanics and spoken of as the laws of *a degenerate electron gas*. According to these laws the average energy of one electron in the gas is given by the expression

$$\frac{3h^2}{40m}\left(\frac{3}{\pi V}\right)^{\frac{1}{3}}=21.6/V_1^{\frac{1}{3}} \text{ electron volts (ev)}, \qquad (2)$$

where V_1 is the volume per electron in cubic angstroms, h is Planck's constant and m the mass of the electron. The energy of an assembly of one gram atom of electrons is $Eg=359/V^{\frac{1}{3}}$ kilocalories where V represents the volume in cubic centimeters of one gram atom. This energy

TABLE I.

		Lattice Constant (angstroms)	Heat of Sublimation (kilocalories/mole)	Compressibility (cm^2/dyne)
Lithium	(calc.)	3.49	34	8.4×10^{-12}
	(exp.)	3.46	39	7.4×10^{-12}
Sodium	(calc.)	4.53	23	12.0×10^{-12}
	(exp.)	4.25	26	12.3×10^{-12}

is the B term in the energy equation. In terms of the energy we can find the pressure of the gas obtaining

$$p=-dEg/dV=9.9\times10^{6}V^{-5/3}\text{ atmospheres,} \quad (3)$$

where V again means the volume of one gram atom of our degenerate gas particles. We see that the degenerate gas behaves very differently from a classical gas in that its behavior is independent of temperature; this is not strictly true: Actually the gas pressure and energy do depend on temperature but only so slightly that we can neglect these effects in our discussion. We see also that the pressure is always positive, meaning that the gas always tends to expand the metal. Since the metal does not fly apart, there must be forces which act to overcome the pressure due to the B term. Such forces are represented by the C term and they arise from the electrostatic interactions that we neglected in the first approximate treatment of the electron gas.

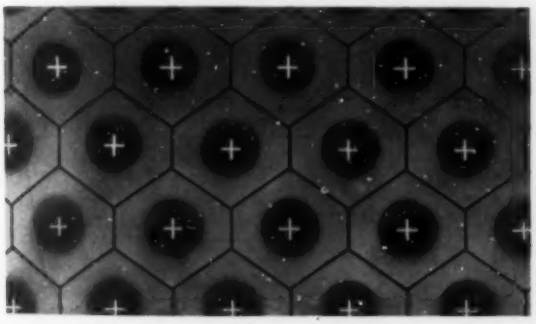


FIG. 4. Division of lattice into neutral polyhedral cells.

Contained in the electrostatic energy are interactions of three types: ion-ion, ion-electron, and electron-electron. In Fig. 4 is shown a mathematical division of the lattice into polyhedral cells with the help of which we shall reduce the consideration of the three types of interaction to a consideration of only the ion-electron interaction within one cell. The cells are constructed by taking one ion and that part of the surrounding electron gas which is nearer to it than to any other ion. Each cell is electrically neutral: the plus charge of the ion being balanced by a compensating volume of the electron gas. Furthermore, the cells are actually rather good approximations to spheres. This is suggested in Fig. 4 and is even more apparent in Fig. 5 where a three-dimensional view is shown. Now a neutral and spherical distribution of electricity exerts no electric field outside its boundaries. Thus, if our polyhedral cells were true spheres they would exert no electrostatic forces on each other and we could calculate the electrostatic energy by considering one sphere alone. Actually the polyhedral cells, not being spherical, do exert small forces on each other; however, when the effect of these forces is determined mathematically, it is found that they are very small indeed and we shall neglect them in what follows.* We therefore consider the electrostatic energy of one polyhedron. As the size of the lattice is reduced, the cells become smaller and the negative electron gas is packed more closely about the positive ion. Since positive and negative electricity attract each other, this means that

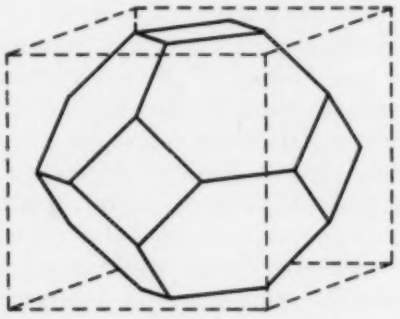


FIG. 5. Three-dimensional view of neutral polyhedral cell.

there is a lowering of potential energy. Since, furthermore, the energy of interaction of two charges varies inversely as the first power of their distance of separation, we find that this energy varies inversely as the lattice constant, which is a measure of how far on the average

* This reasoning suggests that since there is no important force between cells, nothing holds the crystal together and it will fall apart into separate cells. This conclusion is invalid because the electrons are actually crossing over the cell boundaries continually, and if the lattice were to be expanded, no vacant spaces would appear between the cells. As for all cases of lattices in equilibrium, it is not easy to see in detail how the cohesive forces are exerted between the atoms; the usual procedure—and the one adopted in this article—consists of showing that the energy of the crystal as a whole is least for a certain arrangement of the atoms—the equilibrium arrangement. The cohesive forces are then inferred from the increase in energy accompanying any change in the arrangement.

the electron cloud is from the ion. Since the lattice constant is essentially $V^{\frac{1}{3}}$, this interaction gives as a term $-C/V^{\frac{1}{3}}$ in the energy. This term is attractive and tends to reduce the volume; its effect is strong enough to overcome the effect of the electron gas pressure and it is responsible for the binding energy. Under certain simplifying assumptions the theory predicts a value of 673 kilocalories for C.

The A term in the energy cannot be explained simply. It arises from the wave nature of the electron. An electron moving in the field of a sodium ion possesses a certain characteristic wave function and this wave function has, roughly speaking, a certain wave-length. Since the crystal has also a characteristic length, the lattice constant, we may expect an energy term depending upon the relationship between these lengths. This energy term is always positive and tends to expand the lattice.

The same type of wave-mechanical considerations as are necessary in dealing with A play a role in modifying the degree of "freeness" of the electron gas. Failure of the gas to be entirely free manifests itself in the constant B, which may then deviate from the value of 359 kilocalories deduced for the ideal degenerate gas.

We have already referred to the calculations of Bardeen in connection with Table I and Fig. 3. In his work the energy was calculated as a function of the volume and the minimum energy was found to occur for a volume corresponding to a lattice constant of 4.53A. If the metal is compressed by applying an external pressure the energy will be altered. By analyzing the way in which the energy is altered when the volume changes from its equilibrium value of V_0, Bardeen has calculated from his theory the dependence of the compressibility upon the fractional degree of compression, $\Delta V/V_0$, and obtained the results shown in Fig. 6. Experimental curves obtained by Bridgman of Harvard, who used pressures as high as 40,000 atmospheres in this work, are also shown. We see that the agreement between theory and experiment is as good for large degrees of compression as for the equilibrium state. For the other alkali metals, complete calculations are not available, but even so it is possible to compare theory with experiment by determining the constants A, B and C in the energy equation from certain experimental data. Having done this we can then use Eq. (1) to make predictions about other experimental results. The procedure adopted by Bardeen in this case was as follows: the constants A, B and C were determined by requiring (a) that Eq. (1) have a minimum at the observed atomic volume, (b) that the energy given by Eq. (1) at the minimum (i.e., the binding energy) agree with the experimental value and (c) that the compressibility calculated from Eq. (1) (this is given by the curvature at the minimum) agree with the experimental value. These three conditions suffice to determine the three constants A, B and C, and the values so obtained are given in Table II. The comparison between theory and experiment was next made by calculating from Eq. (1) the fractional degree of compression produced by a given external pressure. The results of these calculations are shown in Fig. 7 together with the experimental findings of Bridgman. The agreement is seen to be very good indeed. The goodness of agreement is even more impressive and becomes a greater credit to the validity of Eq. (1) when we realize that all the information used in evaluating the constants corresponds to the state at

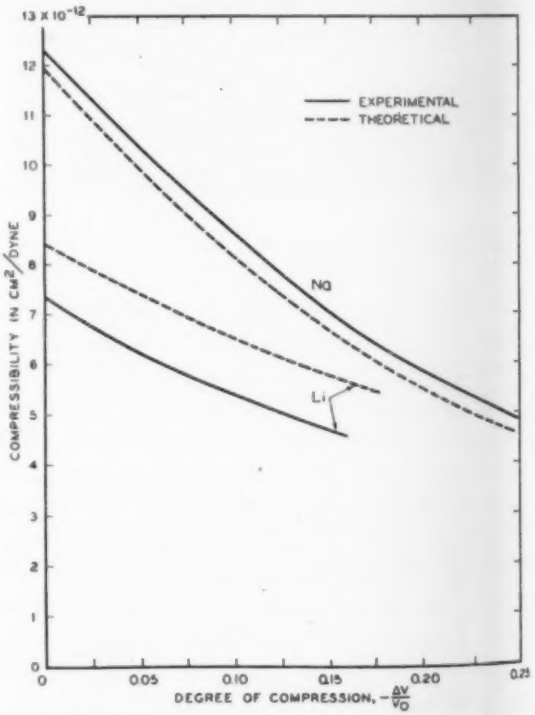


FIG. 6. Compressibility *versus* degree of compression.

zero pressure. This means that Eq. (1) is so good an approximation to the actual energy-volume relationship that we can use it effectively to make extrapolations from the normal state to states of very high pressure where the degree of compression may be nearly 50 percent, as in the case of cesium. Furthermore, the second datum used in determining A, B and C was the binding energy. A little consideration will convince the reader that, so far as experiment is concerned, the connection between the binding energy and the data on Fig. 7 is quite remote; nevertheless the former can be used to determine the constants of Eq. (1); and when this is done, Eq. (1) then gives good agreement on Fig. 7. Of course, both the binding energy and the pressure produced by compression are due to the same cause, the forces of interaction of the electrons and ions. Eq. (1) attempts to relate the various aspects of these forces by giving an explicit relationship between energy and volume, and the success of this attempt over such a wide range of pressures is good evidence for the validity of the new theory and its physical picture.

The picture presented here of the positive ions floating in the electron gas is not, in its general aspects, new. The classical theory of metals developed by Drude and Lorentz also embodied the idea of an electron gas. But the behavior of the electron gas predicted classically was unsatisfactory in many respects: the pressure exerted by it would be too small, its thermal behavior would be incorrect (we shall return to thermal properties below), and the electrons would tend to form clusters about the ions rather than a uniform smear of charge. We shall see below that this uniformity of the charge distribution is a significant feature of the new theory. The essential idea of the older theory, the presence of free electrons, has been preserved in the newer theory; but where questions of a quantitative nature are concerned, the wave-mechanical methods have made drastic changes —changes whose success we may judge from a consideration of the agreement between the new theory and experiment exhibited above in Table I and Figs. 3, 6 and 7.

TABLE II. *Empirical constants for Eq. (1) in units of kcal./mole.*

	THEORY	Li	Na	K	Rb	Cs
A	—	250	1450	3170	3170	3470
B	350	660	142	−373	−121	0
C	673	700	640	570	615	654

The Rigidity of the Alkali Metals

The comparison between theory and experiment has so far been discussed only for the equilibrium state and for uniform compression.

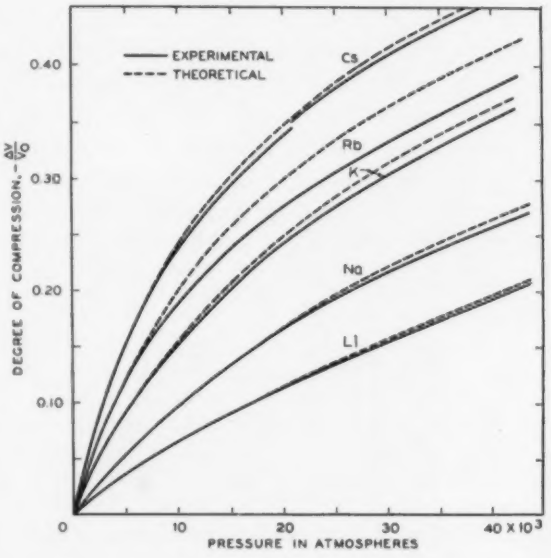


FIG. 7. Degree of compression *versus* pressure.

We must next see how the theory explains the rigidity—that is, the elastic resistance to shear.

The resistance to shear arises simply from the electrostatic repulsions of the ions. The ions tend to get as far apart from each other as possible; however, they are held together by the attraction of the electron gas. This prevents the repulsion from tearing the crystal apart. However, at any particular volume of the crystal, the ions will naturally arrange themselves so as to be as well separated as possible, and this can be shown from theory to result in a regular lattice such as the body-centered lattice. In Fig. 8, we see the effect of shearing the regular lattice, greatly exaggerated. It is apparent from this figure that shear has the effect of bringing the positive ions closer together on the average and this means raising the energy. Hence we should have to do

work and consequently exert forces in order to strain the metal elastically as represented in Fig. 8 and therefore the metal will exhibit rigidity.

Quantitative calculations of these forces can be made using the classical theory of electrostatics. In carrying out these calculations, however, one must make use of the charge density of the electron gas. Assuming that the gas is uniformly distributed—an assumption, we emphasize, that is warranted by the new wave-mechanical calculations and not by the older classical

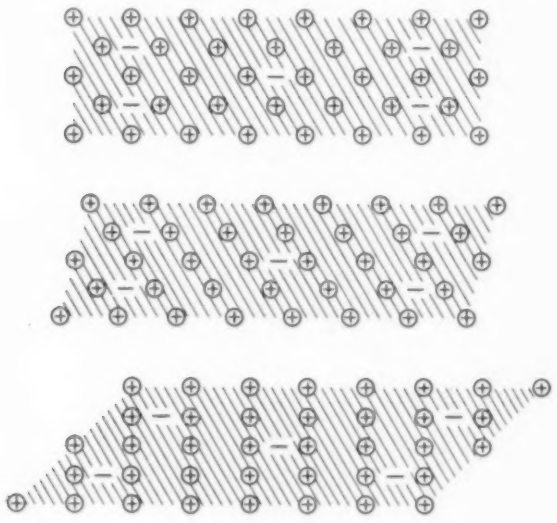


FIG. 8. Effect of electrostatic interactions on shear.

theory—values of the rigidity or shear moduli have been calculated. The rigidity of a nonisotropic crystal is different in different directions; but for a cubic crystal, like sodium, the rigidity is completely specified by giving the rigidity moduli for two different directions. The first two columns of Table III contain the computed and measured shear moduli for sodium. The computed values were obtained by K. Fuchs of the University of Bristol[3] on the basis of the electrostatic theory just described using the uniform distribution of the electron gas; if a non-uniform distribution, like that of Fig. 2(a) for example, had been used, quite different values would have been found. The experimental data were obtained by Quimby and Siegel of Columbia University[4] —after the theoretical results of Fuchs had been published. The satisfactory agreement between theory and experiment is the final piece of evidence we offer in support of the picture based on the new theory.

The last column of Table III and Table IV is given for completeness. The former gives the bulk modulus, $1/K$; it is the reciprocal of the compressibility, which was discussed above. The three elastic moduli given in Table II completely describe the elastic properties of sodium; and from them and the definitions at the heads of the columns, we can find, if we wish to, the so-called elastic constants C_{11}, C_{12} and C_{44}. Table III gives Fuchs' formulae for the rigidity moduli of face- and body-centered crystals obtained by using classical electrostatic theory together with the assumption of a uniform distribution for the electron gas.

The evidence we have presented above in favor of the new theory has been concerned with the simpler properties of the alkali metals; that is, with the energy and its variations for slight elastic deformations. In the interest of compactness, we shall not discuss other evidence connected with the electric, magnetic, and thermal properties which also supports the new theory. Instead we shall assume, hopefully, that the material presented above has been reasonably convincing and shall here merely summarize the results. We have proposed a picture of a metal in which the valence electrons are spread uniformly throughout the crystal constituting a degenerate electron gas. The properties of this gas are specified by certain known quantum-mechanical laws and its energy can be calculated. In this gas the positive ions float freely. The opposing

TABLE III. *Elastic constants of sodium* (dynes/cm²).*

	$C_{11}-C_{12}$	C_{44}	$1/K=\frac{1}{3}(C_{11}+2C_{12})$
Calculated	1.41×10^{10}	5.8×10^{10}	8.3×10^{10}
Observed	1.45×10^{10}	5.9×10^{10}	8.1×10^{10}

* A small correction is made in this table for the overlap interactions between ions, which is explained in connection with Table VI.

TABLE IV. *Formulae for electrostatic part of elastic constants (values are in dynes/cm² and a is lattice constant in angstroms).*

	BODY-CENTERED	FACE-CENTERED
$C_{11}-C_{12}$	$4.56\times10^{12}/a^4$	$9.65\times10^{12}/a^4$
C_{44}	$17.0\times10^{12}/a^4$	$43.3\times10^{12}/a^4$

tendencies of the gas to expand and the ions to hold it together compliment each other and result in the establishment of equilibrium. The ions repel each other and as a consequence of this they are arranged in regular lattice.

The Failure of an Older Theory

The new picture is in disagreement to a large extent with an older picture: the one associated with the phrases "forces between atoms" and "interatomic forces." According to the interatomic force picture, each atom preserves its identity in the solid and attracts or repels the other atoms according to some laws governing the interatomic force. In the newer picture the situation is more complicated. The atoms do not preserve their identity as entire atoms but instead lose their valence electrons to the electron gas. The interactions which arise in this case can hardly be considered "interatomic;" instead there are present forces of a new type such as the pressure of the electron gas. We have seen that the new viewpoint succeeds in explaining the experimental facts, but this is not a sufficient reason for abandoning the older simpler picture; we must next show that the older picture fails.

The failure of the interatomic force picture for the case of metals arises from its effort to explain both the bulk modulus, which involves volume change, and the shear moduli, which do not, with the aid of one and the same set of forces. This idea is illustrated in Fig. 9, where forces between atoms are regarded as acting along the lines. In top portion the lattice is unstressed; in middle it is compressed; in bottom it is sheared. In the stressed cases the same set of forces come into play in both shear and compression. This suggests that since the same interatomic forces determine both the shear moduli and bulk modulus, it should be possible to calculate the latter in terms of the former. The solution of this problem was worked out by Cauchy who found that for cubic crystals the bulk modulus $1/K$ was related to the two shear moduli C_{44} and $C_{11}-C_{12}$ by the equation $1/K=C_{44}+\frac{1}{3}(C_{11}-C_{12})$; this result is quite general so long as the atoms interact with forces directed along the line of their centers; it is not restricted to cases like Fig. 9 where only nearest neighbors interact. In Table V we give the data necessary to test Cauchy's condition for several metals and also for several ionic compounds.[5] It is seen to fail for the metals, the bulk modulus calculated from the shear moduli being considerably smaller than the measured value. The reason for this is clear on the basis of the newer picture: in the case of shear only the electrostatic ion repulsions enter, whereas for compression the electron gas pressure and other forces enter as well, thus raising the bulk modulus. For comparison, values are given for some ionic crystals; for these there is no electron gas and the forces between ions should be of a straightforward interatomic nature, and the Cauchy condition is quite well satisfied. For other crystals where there are directed valence bonds, such as diamond, we should expect that Cauchy's conditions would fail.

TABLE V. *Test of Cauchy conditions (in units of 10^{10} dynes/cm²).*

	$C_{12}-C_{44}$	C_{44}	$1/K_{CALC}=C_{44}+\frac{1}{3}(C_{11}-C_{12})$	$1/K_{MEAS}$	K_{CALC}/K_{MEAS}
Na	1.45	5.9	6.4	8.1	1.27
Cu	51	82	99	139	1.40
NaCl	33.9	12.7	24.0	24	1.00
CaF_2	119	34	74	82	1.10

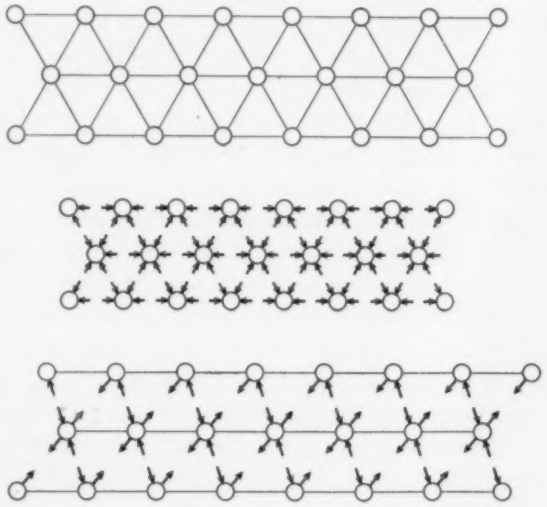


FIG. 9. Illustrating the central force assumption of Cauchy.

Concerning Work Hardening and Thermal Effects

Now that we may regard our electron gas and floating ion picture as being rather well established, let us apply it to a question of metallurgical interest: does cold work with consequent work hardening cause something to happen to the atoms? For example, it has been proposed that the cold working of a metal causes the breakdown of the metallic bond and the formation of homopolar bonds. In other words, are the atoms somehow *changed* in the process so that they exert *different kinds* of forces on each other before and after the cold working?

We shall answer this question in the negative. We consider first of all the electron gas itself. Possessed as it is of a gas-like character, it is evident that it can be given no permanent distortion or alteration. It may, of course, be compressed as in Bridgman's experiments, but this will not alter its character, and as soon as the pressure is relieved it reverts to its normal volume. We must next consider the ions and determine whether anything can happen to them, first individually and second collectively. In discussing the atom, we mentioned that the valence electron may be fairly easily removed but after that it is very much harder to remove another electron from the ion. Even the extreme pressures used by Bridgman barely suffice to make the outermost edges of the ions overlap and by no means push them close enough together to knock off an electron. Before that would happen we should need pressures at least twenty times Bridgman's and even if these pressures somehow managed to come about, in localized regions of stress concentration perhaps, unless they were maintained externally the ions would return to their normal state. There is another possibility: we know that atoms and ions may exist in excited states and when in these, their properties are quite different. However, excitation of the ion of the sodium atom would require pressures of the same order of magnitude as those needed to remove another electron. We may therefore conclude that no amount of cold work will alter the ions and we may continue to regard them as effectively unalterable units of plus charge floating in the electron gas. There is one other possibility, and that is that somehow the relative positions of the ions can be altered to some new and stable pattern. Such a change does apparently take place at the break in the curve for cesium at 20,000 atmospheres: it is believed that, above that pressure, the ions are arranged on a face-centered lattice; however, when the pressure is relieved, they revert to the body-centered lattice. From a knowledge of the x-ray patterns of the most severely cold-worked metals, we find that the atoms are arranged as in samples of well annealed metals except for slight strains. These strains are of an elastic nature and relieve themselves as soon as stresses are relieved by heating. Although these strains are too small to alter in any way the nature of the forces, they do, of course, produce systems of internal stresses in the metal which may, furthermore, hinder the motion along slip planes through the lattice and thus give rise to hardening. Theories along these lines are at present in a very early stage,[6] and we shall not discuss them further. The present reasoning has been based upon consideration of the alkali metals; it is sometimes said that these metals do not show work hardening and consequently a discussion of their properties is not pertinent. The writer disagrees with this view for the following reason: tests made with sodium at room temperature, which is 0.8 times the melting temperature on an absolute scale, would correspond to tests at 800°C or 1500°F for copper and 1200°C or 2100°F for iron. At such elevated temperatures annealing proceeds so rapidly that work hardening cannot be observed. The writer believes that measurements made at liquid nitrogen temperatures would reveal work hardening for the alkali metals. Furthermore, the theoretical arguments advanced for the alkali metals hold for other metals as well. We may conclude then that our picture of the electron gas and floating ions, which explains experiments over a range of pressures much larger than are met with in cold work, precludes the possibility of anything happening to the atoms during cold working; that is, anything of such a nature as to change the types of forces acting in the metal.

So far in our discussion of the theory, temperature has played no role. We have been restricting our remarks to low temperatures

where the stable state of the system is the one with least energy. The thermal theory of metals is in itself a large topic in the theory of solids and we can afford space here only for two of its most important concepts.

When a metal is heated it absorbs energy. This energy, the theory tells us, is used almost entirely for one purpose: it produces vibrations among the ions. At temperatures well below the melting point, the amplitudes of these vibrations are small compared to the lattice constant and each ion stays near its equilibrium position. As we have said before, very large energies are required to alter an ion—so large in fact that no appreciable number of ions would be excited thermally below temperatures of 200,000° centigrade. Thus, except for their vibration, the ions are

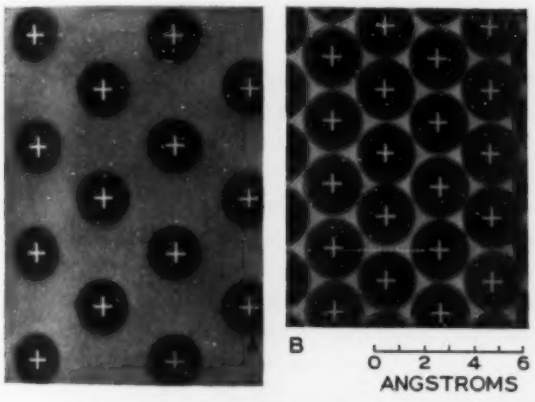


FIG. 10. Electronic distributions in sodium (A) and copper (B).

unaltered by temperature changes. (The ions do not, for example, expand.) Owing to the peculiar nature of the degenerate electron gas, the electrons in it are not appreciably affected by any ordinary temperature changes. Thus, as we have said above, all the energy goes into making the ions vibrate about their equilibrium positions. The theory of specific heats is based on this concept.

The second concept to discuss is that of thermal expansion. Thermal expansion arises from the fact that it is more difficult to compress than to expand a metal. In Fig. 6, we see that the compressibility decreases with compression—that is, the more the metal is compressed, the harder it becomes to compress it further. During the thermal vibrations, various parts of the metal are continually expanding and contracting in accordance with the vibrations of the ions. Since the compressions are more difficult to realize than the expansions, the net effect is that there is, on the average, more expansion going on than contraction. This results on a macroscopic scale in a measurable degree of expansion. The amount of contraction and expansion increases with the temperature and so does the macroscopic expansion. Theories based on this picture are capable of predicting the thermal expansion from the specific heat and compressibility curves, like that of Fig. 6.

The Theory for Other Metals

In the case of the alkali metals the ions are spaced so far apart that they interact with each other only electrostatically. In many other metals this is not true—instead, the ions are packed together so closely that their electron shells overlap and this gives rise to additional repulsive forces. The absence of these forces is one important simplifying feature of the alkali metals. For contrast let us consider the case of copper. In Fig. 10, we compare the 111 plane of copper with the 110 plane of sodium; these are the most densely populated planes for the two metals. We see that in copper the ions are much closer together, so close that an additional energy arising from the overlapping of their electron shells must be considered. This overlap energy can be calculated and combined with the other energies to form a complete picture of the forces involved. In Table VI we give the elastic constants calculated for copper, showing the relative importance of the various terms. It is seen that agreement with experiment is satisfactory and that the largest contributions to the elastic forces

TABLE VI. *Composition of the elastic constants (units of 10^{10} dynes/cm²).*

	Cu		Na	
	$C_{11}-C_{12}$	C_{44}	$C_{11}-C_{12}$	C_{44}
Electrostatic Part	5.7	26	1.43	5.3
Ionic Overlap	45	63	−0.02	0.5
Total	51	89	1.41	5.8
Experiment	51	82	1.45	5.9

arise from the overlap energy of the ion shells. Since the principal forces involved in the elastic constants correspond to the interatomic force scheme, we might expect that the Cauchy condition would be well satisfied for copper. In Table V, however, we see that the failure is worse than for sodium, and an investigation into the theory shows that Cauchy's conditions should be fulfilled only if the ions are in equilibrium under their own forces, as in sodium chloride, but not when they are pressed together by the attraction between the ions and the electron gas as in this case.

The alkalis are the simplest metals to treat theoretically because their ions are so well separated that the overlap energy is very small and because they have only one valence electron per atom; no other metals possess both of these simplifying features. Next to the alkalis, the metals copper, silver, and gold are the simplest for they, like the alkalis, give only one valence electron per atom to the electron gas; and although there is appreciable overlapping and large repulsive forces among their ions, owing to the fact that the ions consist of "closed shells" and are not readily excited, these forces may be calculated relatively simply. For other metals there are more electrons contributed to the electron gas and certain new features not mentioned before are evidenced: the electron gas possesses "critical" concentrations. For the face-centered lattice, theory predicts a critical concentration at 1.362 electrons per atom and for the body-centered lattice the concentration is 1.480. These values constitute a basis for the interpretation of the Hume-Rothery electron concentration rules, which state that the α phase (face-centered) ceases at an electron concentration of 1.36 per atom and the β phase (body-centered) occurs for a concentration of $\frac{3}{2}=1.5$ (or approximately 1.48) per atom. Besides variations in the electron concentration there are variations in the nature of the interionic forces. These forces become much more complicated for elements in the transition series, such as manganese, iron, cobalt, nickel, molybdenum, etc. For the ions of these metals we need forces of a nature somewhere between those of the electron gas, the ion repulsions of copper, and the type of directed valence associated with the tetrahedral structure of carbon compounds. These forces are very important in the transition metals and give rise to their high mechanical strength and also to the phenomenon of ferromagnetism. Although it is of much more importance practically to understand the transition elements than the alkali metals, the presence of these strong interionic forces makes it correspondingly more difficult. However, many important features of the nature of these forces are already understood and the theory is frequently able to offer plausible explanations for observed effects where exact analysis is wanting.

In concluding we may point out that the picture of the metallic state proposed here is endowed with a rather appropriate flexibility. For example, metals and alloys are distinguished from valence and ionic crystals by their capacity to form solid solutions over wide ranges of composition. The present theory explains this by picturing a solute atom as becoming ionized, losing its valence electron or electrons to the electron gas and having its ion substituted for one of the ions of the solvent metal. Unless the solute atom is much larger or smaller than the solvent atom, only a slight disturbance will be caused by its presence. In a valence crystal the situation is quite different; if the foreign atom does not have the proper valence, there will be unpaired valence electrons and a large disturbance in its neighborhood which cannot be relieved, as in the metallic case, by electrons going to the electron gas. This same flexibility of structure suggests itself as an explanation for the plasticity of metals; it can be visualized how slip may occur along a plane of ions in the metal without the production of such a disruptive disturbance as would occur for the case of atoms having directed valences or for the case of ionic structures. Metals having strong interactions between their ions will behave more like valence crystals, whereas the alkalis and the noble metals will be more malleable. As we have suggested there are strong interactions between the ions in the transition elements as typified by V, Cr, Mn, Fe, Ni, Mo, W, and these are the ones which show the greatest strengths and at the same time the least malleability.

The present article makes no pretensions of being a complete review of the present state of

the theory of metals. Many important topics, the thermal properties, magnetic properties, thermionic and photoelectric emission, and especially electrical conductivity have been either unmentioned or discussed most superficially. The main emphasis has been placed upon the comparison of theory and experiment for the alkali metals. The purpose of this has been to clarify and establish a picture of the state of affairs in a metal, a picture which only recently has proved its value in giving accurate quantitative predictions in agreement with experiment and which, it now seems, will form the basis for great additional progress in understanding metals. This is, of course, the picture of the degenerate electron gas in which the positive ions float.

Bibliography

1. For other discussions of the new theory see F. Seitz and R. P. Johnson, "The Modern Theory of the Solid State," J. App. Phys. **8**, 84, 186, 246 (1937).
   N. F. Mott and H. Jones, *Theory of the Properties of Metals and Alloys* (Oxford, 1936).
   J. C. Slater, "The Electronic Structure of Metals," Rev. Mod. Phys. **6**, 209 (1934).
   L. A. Dubridge, "The Physics of Solids," Rev. Sci. Inst. **9**, 1 (1938).
   W. Shockley, "The Quantum Physics of Solids," Bell Sys. Tech. J. **18**, July (1939).
2. J. Bardeen, J. Chem. Phys. **6**, 367, 372 (1938).
3. K. Fuchs, Proc. Roy. Soc. **157**, 444 (1936); **153**, 622 (1936); **151**, 585 (1935).
4. S. L. Quimby and S. Siegel, Phys. Rev. **54**, 293 (1938).
5. These data were obtained from the following sources: sodium from references 2 and 4, copper from reference 3, NaCl and CaF_2 from the Landolt-Bornstein Tables.
6. For a discussion and references on this topic see W. L. Bragg, "A Discussion of Plastic Flow in Metals," Proc. Roy. Soc. **168**, 302 (1938).

Accessories for Spectrochemical Analysis

By Stanley S. Ballard and Paul L. Gow

University of Hawaii and Experiment Station of the Hawaiian Sugar Planters' Association, Honolulu, Hawaii

In our work on the spectrochemical analysis of agricultural materials we employ the direct-current arc method of excitation and use a large quartz Littrow spectrograph. The light from the arc is focused on the spectrograph slit or collimator lens by a quartz condensing lens.

We often find it necessary to cut down the photographic speed of the spectrograph so that long burning times can be used without overexposing the plate. Also, it is sometimes desirable to have the photographic speed greater during one part of the exposure than during another part. For instance, in the detection of volatile elements the first few seconds of exposure are by far the most important, while less volatile elements may not be excited for several seconds. The use of internal apertures fitted over the collimator lens is beneficial in reducing the aberrations of the lens, but is unsatisfactory for controlling speed in that the aperture size cannot be changed during the course of the exposure. An iris diaphragm may be mounted at the condensing lens, but the use of small circular apertures will result in cutting down the effective prism base and hence decreasing the resolving power of the spectrograph. Also, it is rather difficult to change the aperture size of the iris diaphragm rapidly and surely.

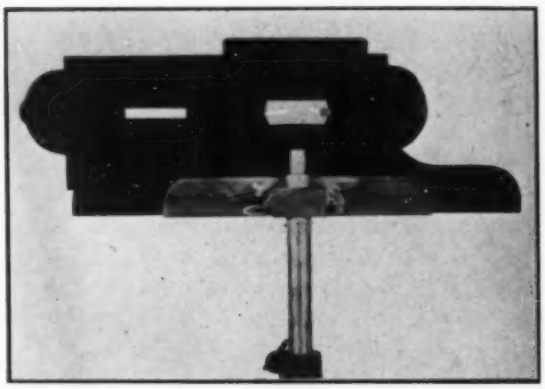


Fig. 1. Device for changing the effective photographic speed of the spectrograph during the course of an exposure. (Mounted on the condensing lens support rod.)

Figure 1 shows a device which we believe overcomes these difficulties. It is patterned after the lantern slide changing device used in projection lanterns. Into each of the two compartments can be slipped a square metal plate in which has been cut a rectangular aperture. The apertures vary considerably in height and hence in area, but have the same width. This width is such that the beam of light entering the spectrograph always fills the collimator lens horizontally and hence gives the maximum theoretical prismatic resolving power. The slide can be thrown to shift from one aperture size to another in a fraction of a second, giving a rapid and sure method of changing photographic speed. As shown in Fig. 1, the device is mounted on the rod holding the condensing lens. The aperture plate shown before the condensing lens in the figure is the one which allows the collimator lens to be completely filled with light, and hence represents the full useful condensing lens aperture. Therefore, there will be a linear relationship between speed and aperture height, when the latter is expressed as a fraction of the height of this particular aperture.

The purity of the arc electrodes is a matter of paramount importance, particularly in qualita-

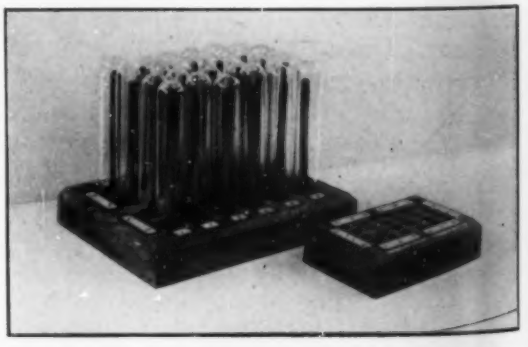
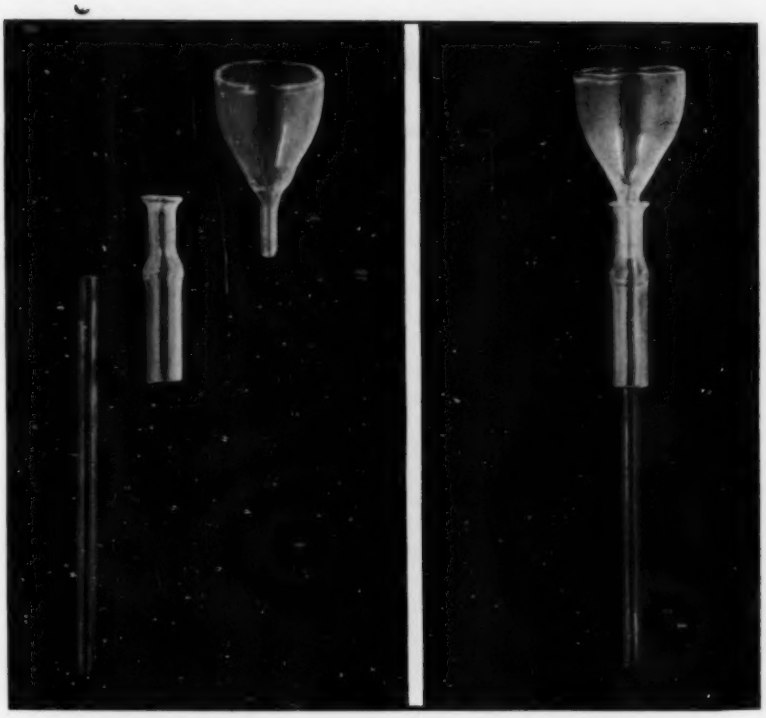


Fig. 2. Protected and unprotected electrode blocks.

tive analysis. We use graphite electrodes of the highest quality obtainable, purchasing them in 12-inch lengths. Each length is broken into two 6-inch pieces, one of which becomes the upper and the other the lower of a pair of electrodes. The ends are turned down in a lathe, using a clean, sharp tool. The upper electrode is rounded on the end, while in the lower electrode a hole of appropriate diameter is drilled to a depth of from $\frac{1}{16}$ to $\frac{1}{4}$ inch. The tool and the drills used are carefully cleaned to remove all oil and grease, are used only in preparing electrodes, and, when not in use, are kept in a desiccator to prevent rusting. While being machined, the electrodes are wrapped in clean filter paper to prevent contact with the lathe chuck. Prepared electrodes are protected from contamination by room dust, etc. by placing them in the larger wooden electrode block shown in Fig. 2. The prepared ends are, of course, at the top, and each electrode is protected by an inverted test tube of the rimless type set in a counter-bored hole centered at the electrode. By the numbers opposite the holes on the block the identity of the electrodes is preserved. After use the electrodes are placed in the proper holes in the smaller block shown in Fig. 2 to await cleaning. After breaking off the arced portions and resurfacing the ends they are again put in the protected block. Since each pair of electrodes is used six or eight times, it is of value to keep a record of the spectrum plates on which they are used, noting any impurities which are detected. This is particularly true since even the purest graphite electrodes contain occasional impurities—impurities which are often elements of interest in analysis (e.g., copper, calcium, magnesium, boron, silicon).



(a) (b)

FIG. 3. Sleeve and funnel device for introducing ground samples into cored arc electrodes. (a) Shown dismantled, and (b) shown assembled on an electrode.

The problem of the quantitative transfer of a weighed sample into the crater of the lower arc electrode has been found troublesome. In the case of finely ground samples such as soils and ashed plant materials this transfer may be readily effected by means of the device shown in Fig. 3. It consists of a small glass sleeve which is slipped over the lower arc electrode and is held in place there by a constriction in its diameter. Into the sleeve is fitted a small glass funnel, the sleeve supporting the lower end of the funnel directly above the pit in the electrode. The sample is brushed from the watch glass upon which it was weighed into the funnel. Tapping the funnel smartly causes all the powder to fall into the pit in the arc electrode. The funnel and the sleeve are then removed, washed and dried, and are ready for re-use.

Here and There

Dr. Alexander J. Allen has been made Assistant Professor of Physics at the University of Pittsburgh and will take up his new work September 1. He is now chief physicist at the Biochemical Foundation of the Franklin Institute.

★

Dr. Forrest F. Cleveland of Lynchburg College, Virginia, has been appointed Professor of Physics at Armour Institute of Technology.

★

At the University of Utah this year Dr. Thomas J. Parmley was promoted from Associate Professor of Physics to Professor and Dr. Irvin J. Swigart was promoted from Instructor to Assistant Professor of Physics.

★

Dr. J. C. Street has been promoted from Assistant Professor to Associate Professor of Physics at Harvard University.

★

The following appointments of men completing the work or the Doctorate in Physics at Yale University have recently been made: E. M. Grabbe has accepted a position with the U. S. Rubber Company in their research laboratory at Passaic, New Jersey; R. F. Humphreys has been appointed an instructor at the University of Syracuse; W. A. Tyrrell, Jr. has accepted a position at the Bell Telephone Laboratories and will be stationed at the Holmdel Branch; J. H. Wakelin has accepted a position with the Goodrich Rubber Company in the Research Department at Akron, Ohio; T. C. Wilson has accepted a position with the New Jersey Zinc Company in the research laboratory at Palmerston, Pennsylvania; G. A. Wrenshall has been appointed an instructor at Hamilton, Ontario.

Dr. W. J. Archibald, who has held an appointment as Sterling Fellow for the past year, has gone to the National Research Council of Canada where he will work with R. W. Boyle. Dr. D. G. Brubaker, who has been a Sterling Fellow for one year, has been appointed an instructor in the Physics Department at Yale University.

★

Dr. J. D. Cockcroft has been appointed Professor of Natural Philosophy at the University of Cambridge, England. Dr. Cockcroft was responsible along with Dr. Kapitza for the design of the equipment used in the production of intense magnetic fields in Cambridge. Recently he has been guiding the construction and development of the Cambridge cyclotron.

Dr. George David Birkhoff has been appointed Harvard Exchange Professor to France for the second half of the academic year 1939–40.

★

Dr. F. A. Scott, former instructor at Lehigh University, has been appointed Assistant Professor of Physics there.

★

Harvard University conferred the degree of Doctor of Science on June 22 on Dr. Percy W. Bridgman, Hollis Professor of Mathematics and Natural Philosophy at Harvard and on Dr. Charles F. Kettering, President of the General Motors Research Corporation.

★

At the annual meeting of the Society for the Promotion of Engineering Education Dr. Frederick L. Bishop of the University of Pittsburgh was presented with an illuminated scroll expressing the Society's appreciation for his twenty-five years of faithful service as secretary of that organization.

★

The Lamme Medal of the Society for the Promotion of Engineering Education was awarded to Dr. Stephen Timoshenko of Stanford University at the annual meeting held this year at State College, Pennsylvania.

★

Professor George E. Uhlenbeck is returning to the University of Michigan as Professor of Physics after an absence of five years as Professor of Physics at the University of Utrecht, Holland.

★

The University of Mississippi has under construction a new physics building and new astronomical observatory. These buildings are being constructed under a P.W.A. grant and represent a total investment of $136,300. The Physics Unit is now nearing completion. The Observatory is under way. The Observatory will have two domes, one 26 feet, the other 13 feet.

★

The Fall Convention of the Electrochemical Society will be held in New York City at Hotel Commodore from September 11 to September 13, 1939.

★

The list of birthday honors of the King of England includes a knighthood conferred on Dr. Owen Richardson, Professor of Physics at the University of London, from 1906 to 1914 Professor of Physics at Princeton University.

★

Dr. R. S. Mulliken of the University of Chicago received the honorary degree of Doctor of Science from Columbia University in June.

Among the recent elections to foreign membership of the Royal Society are the following names: Professor George von Hevesy and Professor Herbert Freundlich.

★

Professor J. H. Van Vleck, of Harvard University, gave a series of lectures at the Institut Henri Poincaré at the University of Paris during May. He also took part in a symposium on magnetism at Strasbourg.

★

At the dedication of the McDonald Observatory Professor Arthur H. Compton, of the University of Chicago, spoke on "The First of the Sciences," and outlined the close relationship between astronomy and physics.

★

Professor P. G. Kruger will be on sabbatical leave during the first semester of the coming year and expects to spend this time at Berkeley, California, working with Professor E. O. Lawrence and the cyclotron.

★

Professor Maurice Ewing has been granted a renewal of his Guggenheim Fellowship for the year 1939–40 to enable him to continue his seismic investigations of submarine geological structures.

★

The ninth annual Convention of the Biological Photographic Association will be held September 14–16, at the Mellon Institute for Industrial Research, Pittsburgh, Pa. The program will be of interest to scientific photographers, scientists who use photography as an aid in their work, teachers in the biological fields, technical experts and serious amateurs. Further information about the Association and the Convention may be obtained by writing the Secretary of the Biological Photographic Association, University Office, Magee Hospital, Pittsburgh, Pennsylvania.

★

Calendar of Meetings

September	
4–12	International Congress of Mathematicians, Cambridge, Mass.
4–15	International Union of Geodesy and Geophysics, Washington, D. C.
11–15	American Chemical Society, Boston, Mass.
20–23	Institute of Radio Engineers, New York, N. Y.
October	
5–7	American Institute of Mining and Metallurgical Engineers, Coal Division, Columbus, Ohio.
12–14	Optical Society of America, Lake Placid, N. Y.
23–25	American Institute of Mining and Metallurgical Engineers, Metals Division, Chicago, Ill.
23–27	American Society for Metals, Chicago, Ill.
November	
2	U. S. Institute for Textile Research, New York, N. Y.
2–4	American Institute of Physics, Symposium on Temperature: Its Measurement and Control in Science and Industry, New York, N. Y.
3–4	Acoustical Society of America, Iowa City, Iowa.
2–4	American Institute of Mining and Metallurgical Engineers, Industrial Minerals Division, Tuscaloosa, Ala.
15–17	American Institute of Chemical Engineers, Providence, R. I.
December	
1–2	American Physical Society, Chicago, Ill.
27–Jan. 2	American Association for the Advancement of Science, Columbus, Ohio.
28–30	American Physical Society, Columbus, Ohio.
28–30	Geological Society of America, Minneapolis, Minn.

New Books

Refraction of the Human Eye and Methods of Estimating the Refraction. Third Edition. JAMES THORINGTON, revised and edited by J. MONROE THORINGTON. Pp. 412, Figs. 283, 15×22 cm. P. Blakiston's Son & Company, Philadelphia, 1939. Price $3.50.

This book is evidently written for the student of ophthalmology. The first five chapters deal with the optical properties of mirrors, prisms, lenses, sphero-cylinder and lens-prism combinations. Then the optics of the eye is discussed, and is followed by methods of visual recording. The principles of retinoscopy are presented in considerable detail, and a section on fitting spectacles and eye-glasses is included.

The reviewer is not competent to judge the soundness of the ophthalmological methods presented, but the fact that this is a third edition shows that the book has been favorably received in this field. However, in regard to fundamental optical principles, the treatment is poor. Inadequate definitions and incorrect statements are very frequent. For example, it is stated that a light ray is always straight, that a convex mirror always gives a virtual image, that the index of refraction may be defined as the reciprocal of the decrease in speed of light as it passes from space into a transparent medium, that crown glass has little dispersive power because it is isotropic, while flint glass is highly dispersive because it is anisotropic. It is asserted that prisms do not produce images, but merely create optical illusions. Such statements are not worthy of a good student in sophomore physics.

It seems that some discussion of the effect of illumination on visual acuity should be included in a work of this kind. It also seems odd that the structure of the retina and the nerve mechanism associated with vision should not receive any attention.

Though the book may be valuable as a handbook dealing with practical lens-fitting, it seems to be decidedly weak in the basic sciences involved.

JOSEPH VALASEK
University of Minnesota

Principles of Electricity and Magnetism. G. P. HARNWELL. Pp. 619+xiii, 16×25 cm. McGraw-Hill Book Company, New York, 1938. Price $5.00.

In this volume the author has made some quite radical and generally refreshing departures from the usual type of treatise on electricity and magnetism. It is the reviewer's belief, however, that most teachers will find the book valuable chiefly as auxiliary reading for the better students. The weaker ones will find it definitely formidable, and most of the better ones will find it difficult even though the author intends that the book shall constitute a second course on electricity following the general physics course.

The author is to be complimented on introducing in some detail material which is usually only mentioned in most texts. This is particularly true of the section on vacuum tubes and tube circuits. Amplifiers and oscillators are treated with sufficient elaboration to give the student some working knowledge of them. The reviewer feels that more space might have been allotted to a discussion of applications of magnetic and electrostatic focusing of electron beams to electron optical devices, inasmuch as this subject will probably be of increasing importance in the near future. However, with a wealth of material from which to choose, some parts must be slighted.

The practical system of units is used throughout, and the definitions are based on the meter, kilogram and second as fundamental quantities. As far as possible the vector notation is used. In order to familiarize the students with this an appendix on vector theory is attached.

The book is a dignified and worth while addition to the *International Series*. The binding is the same as the other volumes in the series. The type is good and the paper gives only a slight glare. The figures have in some cases been reduced to the point where they seem insignificant.

H. B. WAHLIN
University of Wisconsin

Spontaneous Fluctuations of Voltage. E. B. MOULLIN. Pp. 251+vi, Figs. 88, 16×24 cm. Oxford at the Clarendon Press, Great Britain, 1939. Price $6.

This book describes many phases of the study of voltage disturbances arising from thermal and shot effects, especially in connection with the bearing of these problems on vacuum tube theory and practice. In an introductory chapter of general interest, the character of thermal fluctuation voltages is discussed. Disturbances of thermionic origin are then treated, although not with a satisfying finality, a defect which reflects the lack of completeness in the theoretical development of this subject in the literature. The book includes, for reasons of background, a rather detailed consideration of conduction phenomena in a two-element vacuum tube which may be of interest to physicists quite aside from its application here.

The more involved case of shot effect in space-charge limited conduction, together with fluctuation anomalies such as the flicker effect of J. B. Johnson, disturbances due to residual ionization, secondary emission, and positive ion emission, are described for their bearing on practical problems. The treatment of multi-electrode tubes may be somewhat too brief to satisfy the practical needs of vacuum tube engineers, but this group will find much of interest in a final chapter on technical problems involving voltage fluctuations where the relative importance of thermal and shot disturbances in rectifier and amplifier circuits is discussed. Of practical interest, also, is the description of some work on disturbances due to resistance elements and carbon microphones.

The author gives fairly complete bibliographical references to original source material, although in some cases he has apparently omitted to do so, as in the case of Nyquist's theorem, which is repeatedly mentioned in the text. The organization of the material is in most cases adequate, although any deficiencies in this respect may be attributed to the rather recent dates of much of the reference material.

The book will undoubtedly be of much interest to engineers and physicists, and the author is to be congratulated for his success in bringing together the results of the numerous investigations in this field in which Schottky, Johnson, Hull and N. H. Williams pioneered, and in which the author and his colleagues have made valuable contributions.

C. G. SUITS
General Electric Research Laboratories

Properties of Glass. G. W. MOREY. Pp. 561, 15×23 cm. Reinhold Publishing Corporation, New York, 1938. Price $12.50.

Although the making of glass is an ancient art, it is only within the last decades that a science of glass has developed. Today there is a widespread interest among glass technologists, chemists and physicists in making careful studies of the various physical properties of glass, in applying the new science to the improvement of glass, and in correlating the physical properties and building theories of the constitution of glass.

The present book will be welcomed by all workers in this field, for it brings together a vast amount of empirical information of the greatest importance, but extremely difficult to find. The author of this book has long been associated with the very best of American research in the field of glass and silicate chemistry and is eminently qualified to write the present monograph. The book presents in fairly complete manner the empirical facts and the correlations and theory which seem well established. Theories and speculations too new to be considered as established are mentioned briefly. It is obvious that many topics must be omitted. The book deals solely with inorganic glass. Very little is said about glass making practice, and topics such as glass blowing and glass-to-metal seals are omitted. The scope and contents are best given by the following abbreviated chapter headings: History, Devitrification, Composition, Chemical Durability, Viscosity, Annealing, Surface Tension, Heat Capacity, Heat Conductivity, Density, Coefficient of Expansion, Elastic Properties, Strength, Thermal Endurance, Hardness, Optical Properties, Electrical Conductivity, Dielectric Properties, Magnetic Properties, and The Constitution of Glass.

It is safe to predict that this book will be read by all workers who have any interest in the science of glass and that it will be the standard reference work in this field for many years to come. The book can be highly recommended.

B. E. WARREN
Massachusetts Institute of Technology

Theory and Applications of Electron Tubes. HERBERT J. REICH. Pp. 670+xviii, Figs. 510, 16×23 cm. McGraw Hill Book Company, Inc., New York City, 1939. Price $5.00.

Until recently there were but few authoritative texts in this rapidly growing field. Now, however, the condition is being remedied, five having appeared within three years. Dr. Reich's book is a comprehensive work of reference, with adequate emphasis upon the noncommunication uses of electron tubes. It is soberly written in engineering English, unadorned by picturesque phrases and unmarked by obvious predilections; the author gives his own extensive researches but scant notice. The mathematics, though freely used, is not beyond the comprehension of a first-year graduate student in physics or electrical engineering.

The text is logically arranged, beginning with a chapter on physical concepts and continuing with one on thermionic emission. The author is especially careful to show the transition from theoretical principles to practical applications. A great many circuits and devices are described in sufficient detail to enable the reader to construct them. Television is the only major subject not treated. Particularly noteworthy are the chapters on vacuum-tube oscillators, electrical conduction in gases, glow- and arc-discharge tubes and circuits, and electron-tube instruments and measurements. There is an extensive and well-chosen series of references. Most chapters conclude with problems.

Much more than the usual care has evidently been taken in preparation and proof-reading; misprints and misstatements are conspicuously absent.

As a classroom text this book is best adapted for use in an institution where the students are well selected and not afraid to work. There is enough material for at least two courses.

As a reference text it deserves a place on every shelf. The technically trained reader will find in it not only a comprehensive account of the multifarious uses of vacuum tubes, but also a logical and well-knit presentation of the underlying theory, so that he may make new applications with confidence that they are correct in principle and free from the characteristic faults that so often mar the work of the "handbook engineer."

CHAS. WILLIAMSON
Carnegie Institute of Technology

The Perception of Light. W. D. WRIGHT. Pp. 100, Figs. 49, 12½×19 cm. Blackie and Son, Ltd., London and Glasgow, 1938. Price 6 shillings net.

Dr. Wright is Lecturer in Physics at the Imperial College of Science and Technology in London, and a productive worker in physiological optics. He offers this little book as "in the nature of an essay, in which I have tried to analyze the visual phenomena that seem to me to be of most importance to those concerned with lighting problems and with the technical manipulation of visual sensations."

In 95 text pages, Dr. Wright presents a critical review of selected and for the most part current physiological data and theories, and certain of their practical implications, emphasizing what appears to him most interesting and fruitful, and stating frankly his preferences and predilections. The result might be described as an informal commentary on physiological optics, always intelligent and stimulating, and for the most part highly readable.

For workers in visual physiology, the book is primarily a review and rearrangement of material already reasonably familiar. Dr. Wright is strongly impressed with the enormous complexity of the visual processes and openly distrusts present attempts to rationalize them. In the light of his otherwise consistent empiricism, it is curious to find in Chapter 5 a courageous geometrical treatment of the visual sensations themselves. This flight into the unknown—and as Dr. Wright states, unknowable—is illustrated with a number of hypothetical graphs whose form it would be difficult to question since they entirely lack numerical ordinates.

The lighting engineer will find in this book a stimulating analysis of the many physiological variables which modify visual performance. Little of this material attains the state of directly applicable practical information, probably because the practical visual problem encompasses a wide range of psychological and social as well as physiological factors. There is designedly a wide gap between the simple and abstracted experimental arrangements which yield exact data in the laboratory, and the usual conditions and objects of visual regard. It is improbable therefore that much practical information on lighting problems can be synthesized successfully from the data of physiological optics alone. From the practitioner's viewpoint, a direct frontal attack on such problems, such as Luckiesh and Moss have recently described, is probably more fruitful.*

GEORGE WALD
Harvard University

* M. Luckiesh and F. K. Moss, *The Science of Seeing* (D. Van Nostrand Company, New York, 1937).

New Booklets Received

GM Comments, April, 1939. This eight-page booklet describes many of the products of the GM Laboratories such as galvanometers, rheostats, exposure meters, foot-candle meters, illumination control equipment, galvanometer and other types of relays.

The Roller-Smith Company, Bethlehem, Pennsylvania, has recently issued new catalogs on turblator oil circuit breakers, hipot insulator and bushing tester, hipot potential indicator, hipot insulator tester, and type C control relays.

Industrial Instruments, Inc., 162 West 23rd Street, Bayonne, New Jersey, has a new four-page circular on the conductivity bridge.

The Ward Leonard Company of Mt. Vernon, New York has a descriptive circular on a new safety relay which has been developed for installation on amateur radio transmitters to protect the operator against shock from static charges. The relay automatically discharges the filter condensers after each transmission.

Contributors to This Issue

Ernest J. Abbott is President of the Physicists Research Company of Ann Arbor, Michigan. He received his B.S.E. in Electrical Engineering in 1924; his Ph.D. in Physics in 1932, and professional degree of E.E. in 1935, all from the University of Michigan. From 1925 to 1935, he was engaged as research physicist at the University of Michigan on industrial physics problems. In 1935, he organized Physicists Research Company to undertake jobs which do not respond to ordinary methods of treatment, but which can be solved by practical combinations of engineering and unusual physics tools.

Imre Brody received his Ph.D. in 1918 from the University of Budapest, Hungary, where he was assistant in physics from 1914 to 1919. From 1920 to 1922 he worked at the Universities of Frankfurt and Göttingen, Germany. Since 1923 he has been physicist with the Tungsram Research Laboratory of the United Incandescent Lamp Company in Ujpest, Hungary. He has contributed many papers on the quantum theory, theory of solids, physical chemistry.

W. A. Bruce received his B.S. and M.S. degrees from the University of Kentucky in 1932 and 1933, respectively. From 1933 to 1935 he was with the Ashland Refining Company; from 1935 to 1936 he was research assistant to Professor G. E. M. Jauncey at Washington University, Saint Louis; from 1936 to 1938 he was a fellow at the Washington University where he received his Ph.D., and since 1938 he has been with the Carter Oil Company of Tulsa, Oklahoma.

James E. Field received his B.S. degree from Heidelberg College in 1927. After obtaining his M.S. degree in 1929 from the University of Michigan, he was appointed assistant professor of physics at Battle Creek College. He resumed his work at the University of Michigan in 1932, where he received his Ph.D. degree in Physics in 1936. Since 1935 he has been employed by the Research Division of the Goodyear Tire and Rubber Company at Akron, Ohio.

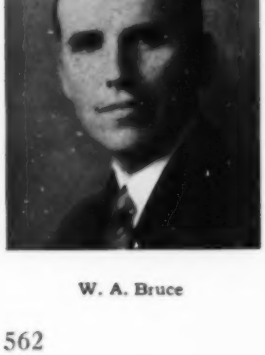
S. D. Gehman received his B.A. and M.A. degrees from the University of Pennsylvania in 1924 and 1925, respectively. He taught physics for a year at Emory University, Atlanta, Georgia, returned to Pennsylvania as a Harrison Fellow in Physics and received his Ph.D. in 1928. Since then he has been with the Goodyear Tire and Rubber Company at Akron, Ohio, as a research physicist.

Francis de Kőrösy, born in 1906 in Budapest, Hungary, graduated in Karlsruhe, Germany, in 1927 as Dipl. Ing. Chem. In 1929 he received his Ph.D. in Budapest. He was assistant in the petroleum and in the textile chemistry laboratory of the Technische Hochschule Karlsruhe until 1931. In 1932 he joined the research laboratory of the United Incandescent Lamp Company in Ujpest, Hungary.

William Shockley's picture and biography appeared on p. 630 of Vol. 9, No. 10, 1938.

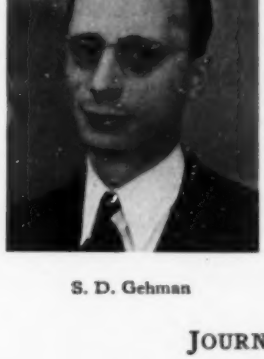
E. J. Abbott

I. Brody

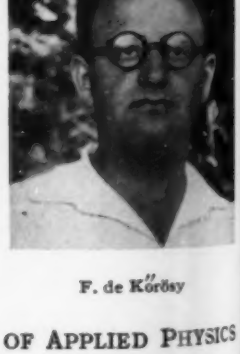


W. A. Bruce

J. E. Field



S. D. Gehman



F. de Kőrösy

In response to the request of the Editor for photographs the above has been sent. We are unable to recognize it but hope that some of our readers can help us out of the difficulty.

Arthur B. Lewis received his B.A. and M.A. degrees from the University of Mississippi in 1923 and 1925. In 1930 he received his Ph.D. degree in Physics from The Johns Hopkins University. From 1926 to 1936 he was with the National Bureau of Standards in the Electrical Division, Section of Electrical Measuring Instruments. Since 1936 he has been Associate Professor of Mathematics and Physics at the University of Mississippi.

Margaret B. Hays received her A.B. degree from Oberlin College in 1924, and her M.S. degree from the University of Pittsburgh in 1925. She was a Fellow in Physics at Bryn Mawr College from 1927 to 1929. In 1929 she accepted a position as Assistant Physicist in the Bureau of Home Economics and since 1935 she has been Associate Physicist there.

Innovations in Instruments

A Television Film Scanner

The Columbia Broadcasting System reports that Dr. Peter Goldmark of their laboratory has devised a new method for the televising of motion picture films. In the ordinary film scanner the film moves in jerks and the scanning is done only while the picture is stationary. In the new method the film passes continuously downward before a scanning lens while an electronic scanning beam moves upward at exactly the same speed so that a stationary electronic image results. A slotted rotating disk is placed between the film and a number of lens segments. This acts as a shutter and gives light to only one of the segments at a time. The result is that sixty separate stationary frames per second can be produced from film which was originally photographed at 24 frames per second, although the speed of action on the receiving screen is not changed in the least. Moreover the received images will have even illumination and great contrast and character. The film scanner has no moving optical parts, uses an incandescent bulb instead of an arc for a light source, is comparatively inexpensive to build, and keeps all its mechanical parts moving at a constant speed.

Electrolytic Conductivity Measurements

A new 40-page illustrated catalog on "Apparatus for Electrolytic Conductivity Measurements in Laboratory and Plant" describes and lists the latest forms of Leeds & Northrup apparatus for making all well-established electrolytic conductivity measurements. It contains a general discussion of research, routine and industrial measurements and includes information on recent developments in technique and apparatus for precise measurements. As a guide in assembling appropriate groups of apparatus for research, general laboratory use, routine testing and educational work, suggested combinations of equipment are illustrated and listed. To receive a copy, ask Leeds & Northrup Company, 4934 Stenton Avenue, Philadelphia, Pennsylvania, for Catalog EN-95.

Megabridge

A megabridge for rapid measurements of leakage resistance has been announced by the Industrial Instruments, Inc., 162 West 23rd Street, Bayonne, New Jersey. The

megabridge is an adaptation of the Wheatstone bridge using an electron ray null indicator. It may be obtained in several ranges from 100,000 ohms up to 100,000 megohms—accuracy is approximately 5 percent.

Contributed Original Research

An X-Ray Investigation of Crystallinity in Rubber

S. D. Gehman and J. E. Field
The Goodyear Tire and Rubber Company, Akron, Ohio
(Received January 17, 1939)

X-ray diffraction evidence has shown that a crystalline structure can be produced in rubber by stretching or by freezing. In the former case, a fiber diagram is generally secured, in the latter, Debye-Scherrer rings. When raw rubber was stretched to moderate elongations and frozen an intense fiber diagram was found, showing that the crystallization proceeded from nuclei set up by the stretching. A series of diffraction patterns illustrating the effect are reproduced. The geometrical conditions of stretching under which "higher orientation" occurs in stretched rubber were studied by photometric measurements of the relative densities of the first two equatorial spots. Graphs are included demonstrating the effect of variations in gauge, width, length and elongation of the specimens. Higher orientation occurs when the percent contraction in gauge exceeds the percent contraction in width. The different physical structures of vulcanized pure gum stocks became apparent in the "higher orientation" characteristics, although the same diffraction pattern was secured. A correlation of the results with current views on the micellar or secondary structure of rubber and the crystallization of supercooled liquids is attempted.

Introduction

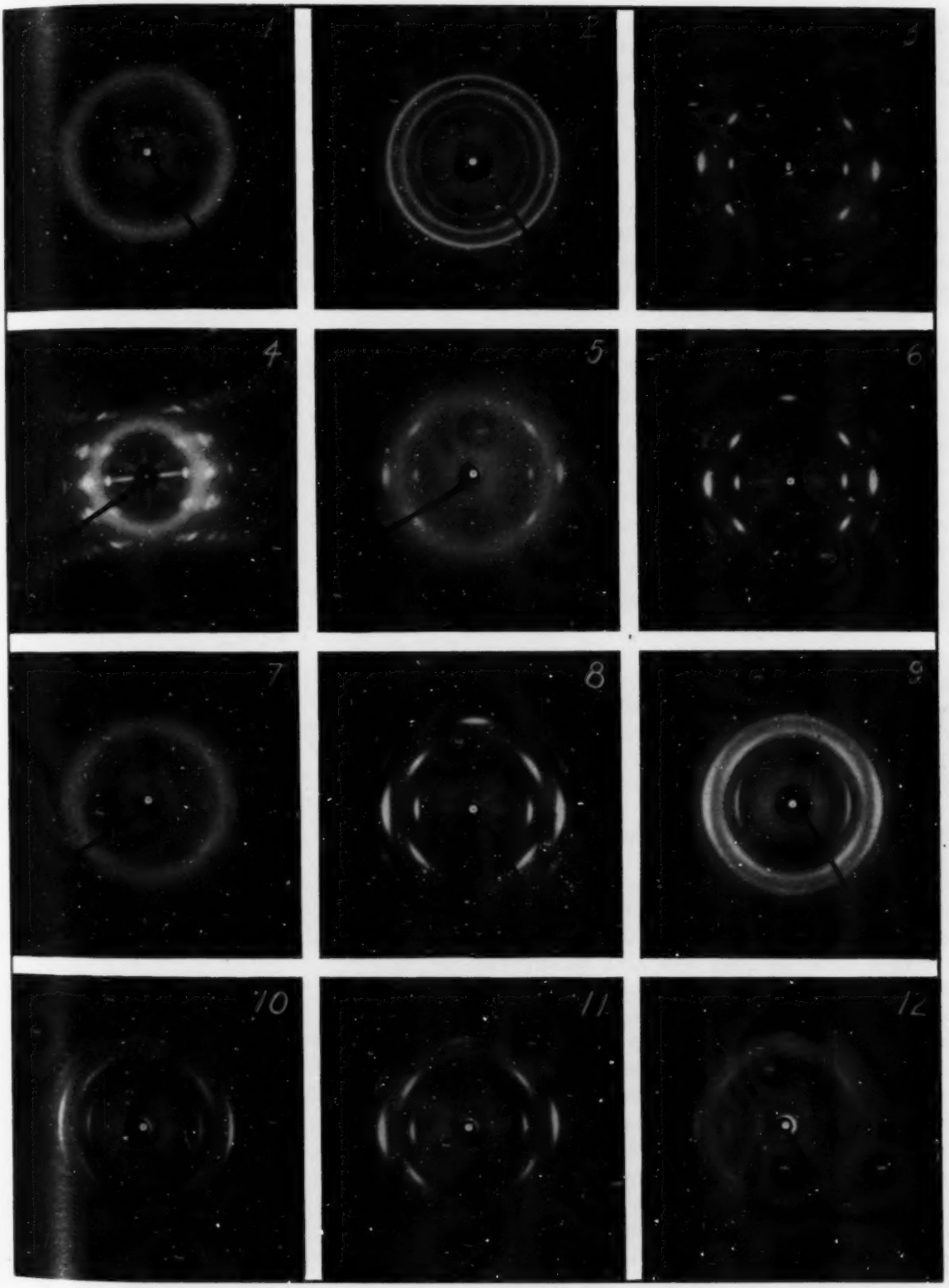
THE crystalline structure which rubber exhibits under certain circumstances[1–3] has come to be regarded as associated with a secondary or micellar structure of long chain molecules. The exact mechanism by which the localized ordered regions appear is a speculative subject in recent developments of the micellar theory of long chain polymeric materials. The views of various workers on this subject have been summarized by other authors.[4–6]

The crystalline structure of rubber displays varying degrees and types of orientation of the crystal units depending upon the conditions under which crystallization occurs. The amorphous x-ray diffraction pattern of unstretched rubber is shown in Fig. 1; the unoriented crystalline diagram for frozen rubber in Fig. 2. When the crystallization is induced by stretching, the crystallites are aligned along the axis of stretching, giving the fiber diagrams of Figs. 3 and 4. In this case, there is random orientation of the other two axes of the crystallites. "Higher orientation," in which all three axes of the crystallites are aligned, gives the diagram of Fig. 15 and can be secured with suitable dimensions of the stretched piece.

The combined effect of stretching and freezing rubber was tested by x-ray diffraction shortly after the fact was discovered that rubber gave a fiber diagram upon stretching. Thus, Hauser and Mark[2] stretched a piece of raw rubber to 500 percent extension, cooled it to −4° C and secured the x-ray diffraction pattern at this temperature. The result of this experiment was negative in that no alteration of the pattern, due to the cooling, was apparent. Hünemörder and Rosbaud[3] obtained a fiber diagram by carefully stretching frozen rubber. Meyer, Susich and Valko also have reported results on the stretching of frozen rubber.[7]

More recently, Smith and Saylor[8, 9] have studied a phenomenon which they have termed the secondary increase in length of stretched, chilled rubber. They observed that raw rubber

[1] J. R. Katz, Naturwiss. **13**, 410 (1925).
[2] E. A. Hauser and H. Mark, Koll. Chem. Beih. **22**, 63 (1926).
[3] M. Hünemörder and P. Rosbaud, Kautschuk **3**, 228 (1927).
[4] A. Frey-Wyssling, Koll. Zeits. **85**, 148 (1938).
[5] O. Kratky, Kolloid Zeits. **84**, 149 (1938).
[6] C. J. B. Clews and F. Schoszberger, Proc. Roy. Soc. **A164**, 491 (1938).
[7] K. H. Meyer, G. v. Susich and E. Valko, Kolloid Zeits. **59**, 208 (1932).
[8] W. H. Smith and C. P. Saylor, Science **85**, 204 (1937).
[9] W. H. Smith and C. P. Saylor, Nat. Bur. Stand. J. Research **21**, 257 (1938).



FIGS. 1 to 12. (1) unstretched rubber, (2) frozen rubber, (3) stretched rubber. 500 percent elongation; (4) racked rubber. 2000 percent elongation; film-specimen distance 3.4 cm; (5) pale crepe rubber. 350 percent elongation; room temperature; (6) pale crepe rubber. 350 percent elongation; frozen; (7) pale crepe rubber. 250 percent elongation; room temperature; (8) pale crepe rubber. 250 percent elongation; frozen; (9) pale crepe rubber. 30 percent elongation; frozen; (10) milled, compounded rubber. 200 percent elongation; frozen; (11) vulcanized rubber. Cure: 40 minutes at 260°F. 200 percent elongation; frozen; (12) vulcanized rubber. Cure: 240 minutes at 260°F. 200 percent elongation; frozen.

stretched to intermediate elongations increased about four percent in length when chilled at about 0°C. This effect was explained as due to a movement of the molecules into the crystalline arrangement, the more parallel arrangement resulting in the increase in length, a reasonable and logical explanation. The present authors have found that the x-ray diffraction phenomena which accompany this effect are much more striking than would be anticipated from the relatively small increase in length as compared to the elongations usually necessary to give rise to a fiber diagram. The results should have a direct bearing on such questions as to whether the rubber crystallites are brought into being or merely made evident by stretching, whether they grow in size or in number as stretching proceeds and how the intensity and size of the diffraction spots are related to the size, number and perfection of the ordered regions responsible for them.

As another possible means of securing new information about the crystallization process due to stretching, the work was extended to investigate more systematically than has been done heretofore the conditions under which "higher orientation" or the ordering of all three axes of the crystallites occurs instead of the parallel alignment of only one axis as in the ordinary fiber diagram.

Apparatus and Methods

The x-ray diffraction patterns were obtained with a General Electric X-Ray Diffraction Unit employing a Philips Metalix copper target tube operated at 35 kv peak and 27 milliamperes. Nickel foil was used as a filter to remove the CuK_β radiation. Film-specimen distance in all cases, except where noted, was 5 cm. Exposure times were usually two or three hours.

Since the intensity of the diffraction pattern for rubber has been shown to depend upon the rate of stretching,[3] the rubber samples, described more specifically later, were stretched to the desired elongations by means of a falling weight. This insured comparable and rapid speeds of stretching. The stretched pieces were clamped to aluminum strips and frozen in the freezing compartment of an electric refrigerator, temperature approximately −5°C, for a period of about 18 hours.

To keep a specimen frozen during exposure, a yoke was made from a thick copper bar. The ends of the yoke were immersed in a mixture of acetone and solid carbon dioxide and the exposed surface insulated with asbestos. A hole was drilled in the center of the bar, suitably positioned for containing the sample and closed by a screw. A one-eighth-inch diameter hole was drilled at right angles to this to permit the passage of the x-ray beam through the sample. The ends of this hole were covered with Cellophane which was kept free of frost by a trickle of acetone over the surface.

The samples were cut with a scissors from the stretched, frozen strips of rubber immersed in acetone-solid carbon dioxide mixture and could be quickly transferred therefrom to the cavity in the copper bar without thawing.

It is now generally known that if raw rubber is suddenly chilled in liquid air or even in an acetone-solid carbon dioxide mixture, it does not give an x-ray diffraction pattern of crystalline material, but an amorphous halo.[10] To secure the Debye-Scherrer rings of Fig. 2, cooling at a moderately low temperature over a period of time is required. After the rubber is frozen in this fashion, it can be chilled at lower temperatures, as in the technique just described, without materially affecting the x-ray diffraction pattern.

In the study of the "higher orientation" in stretched samples, the photometric measurements of the density of the diffraction spots were made with a Zeiss Spectrum Line Photometer.

Results

X-ray diffraction effects due to stretching and freezing

Figure 5 is the diagram secured for a sample of pale crepe rubber stretched to 350 percent elongation, taken at room temperature. Fig. 6 is the diagram for this identical sample when frozen. The amorphous halo has practically disappeared and the diffraction spots are very intense. There is no trace of the Debye-Scherrer

[10] J. R. Katz, Trans. Faraday Soc. **32**, 77 (1936).

rings of Fig. 2. The localized ordered regions or crystallites caused by the stretching act, to all appearances, like nuclei for crystallization when freezing occurs. Figs. 7 and 8 demonstrate the effect for an elongation of 250 percent. The orientation after freezing is less perfect than at the higher elongation, as shown by the greater length of the spots. Fig. 9 is the diagram from a piece of this same rubber stretched to 30 percent elongation and frozen. At room temperature, no diffraction spots were discernible at elongations below 100 percent. In Fig. 9, the Debye-Scherrer rings have strong localized maxima, indicating a transition stage between oriented and random crystallization.

X-ray effects due to the combination of stretching and freezing can also be observed with some vulcanized rubber stocks although the general result of vulcanization is to inhibit or prevent the appearance of the phenomena altogether. Positive results were secured with the following compound:

	Parts by Weight
Pale crepe rubber	100
Zinc oxide	2
Sulfur	1.5
Mercaptobenzothiazole	.5
Stearic acid	.5

Figure 10 shows the diagram for the uncured, milled compound frozen with 200 percent elongation. Strong fibering is evident. This uncured stock, due to the milling necessary for mixing, gave no indication of a fiber diagram at this elongation at room temperature. In fact, the diffraction spots were extremely weak at the highest elongation attainable. Fig. 10 shows that there is sufficient structure left in the milled compound to get, by stretching, effective nuclei for an oriented crystallization at lower temperatures. It is also a fact that extreme breakdown due to heat and milling does not prevent the appearance of the Debye-Scherrer rings due to freezing.

Figures 11 and 12 illustrate that a forty-minute cure of this compound at 260°F does not prevent the appearance of a fiber diagram due to stretching and freezing whereas a longer cure, 240 minutes at 260°F, causes the pattern to become very weak. The minimum elongation for diffraction spots at room temperature was 300 percent.

Under proper conditions of stretching, which will be discussed in the next section, it is possible to freeze rubber with "higher orientation." Fig. 13 is such a diagram for a frozen specimen, which, at room temperature, at this elongation, gave only a feeble diffraction pattern.

Fifteen or twenty exposures were taken with 0.039-inch gauge evaporated latex sheets to study changes during freezing in the patterns of samples stretched to 200 percent and 300 percent elongation, with and without higher orientation. The patterns did not seem of sufficient interest to measure or reproduce here but the following qualitative observations are worthy of note. The alignment in the direction of the fiber axis is less perfect as the freezing progresses. Especially in the case of higher orientation, the spots become somewhat longer. The reverse case, stretching rubber at a lower temperature, is known to give longer spots.[7] The extent of the higher orientation, as judged by the relative density of the equatorial spots, does not appear to change much during freezing. The x-ray pattern approximates its full intensity in a much shorter time than is required for the secondary elongation to approach its limiting value.

Occurrence of higher orientation in stretched rubber

Higher orientation in stretched rubber was first observed by Mark and von Susich,[11] who obtained information necessary for the deduction of the unit cell. In their work, the phenomenon was shown for a thin stretched film and the impression persists in the literature that thin films are necessary to show the effect. Sauter,[12] however, secured higher orientation in racked rubber with a final thickness of one millimeter. There appeared to be an opportunity, by a quantitative study of the conditions of stretching necessary for the effect, to secure information in regard to the nature, perhaps the shape, of the crystallites or ordered regions responsible for the diffraction. The results follow.

Figures 14 and 15 are typical diagrams secured from stretched rubber showing higher orienta-

[11] H. Mark and G. v. Susich, Koll. Zeits. **46**, 11 (1928).
[12] E. Sauter, Zeits. f. physik. Chemie **B36**, 405 (1937).

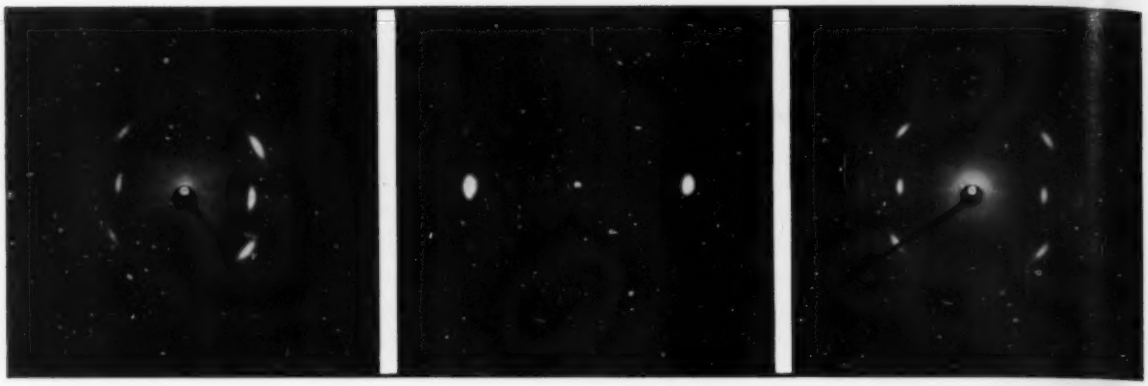


FIG. 13. Evaporated latex. Elongation: 300 percent, frozen. Sample 3 in. wide; 1 in. long.

FIG. 14. Evaporated latex. Elongation: 900 percent, room temperature. Length: 1 inch; width: 3 inches; gauge: 0.039 inch. Beam parallel to surface.

FIG. 15. Evaporated latex. Elongation: 900 percent, room temperature. Length: 1 inch; width: 3 inches; gauge: 0.039 inch. Beam perpendicular to surface.

tion. For Fig. 14, the path of the beam was essentially parallel to the plane of the stretched sheet; for Fig. 15, the beam was perpendicular to this plane. By comparison of Figs. 15 and 3, it is apparent that the relative intensity of the two innermost equatorial spots of the diagram affords a measure of the extent of the higher orientation. In the conventional notation, the first equatorial spot from the center is designated as A_1 and the second spot as A_2. An extensive series of measurements was made of the relative photographic densities of these spots under different conditions of stretching, the direction of the beam always being perpendicular to the plane of the stretched sheet. Figs. 15, 16 and 17 were selected for reproduction to illustrate the changes in the relative intensities of A_1 and A_2 due to a systematic change in one variable in the stretching process, in this case, the distance between grips. The rubber sheets for all this work were made by spreading appropriate thicknesses of ammonia preserved latex on glass plates and drying at room temperature.

The higher orientation was found to depend upon the geometrical variables in the stretching process, i.e., the width, length and other dimensional changes of the test piece due to stretching. Data illustrating this is plotted in Figs. 18, 19, 20 and 21. Instead of plotting directly the photographic density ratio of the A_1 and A_2 spots, the ratio has been used to calculate the percent of the diffracting crystallites which have their c axes essentially parallel to the width of the sheet. This has the advantage of making the data more readily comprehensible but the disadvantage of linking it with the structure of the unit cell, which cannot be regarded as finally determined.[12–14] It is always possible to calculate back to the photographic density ratio

[13] W. Lotmar and K. H. Meyer, Monatsh. **69**, 115 (1936).
[14] H. A. Morss, J. Am. Chem. Soc. **60**, 237 (1938).

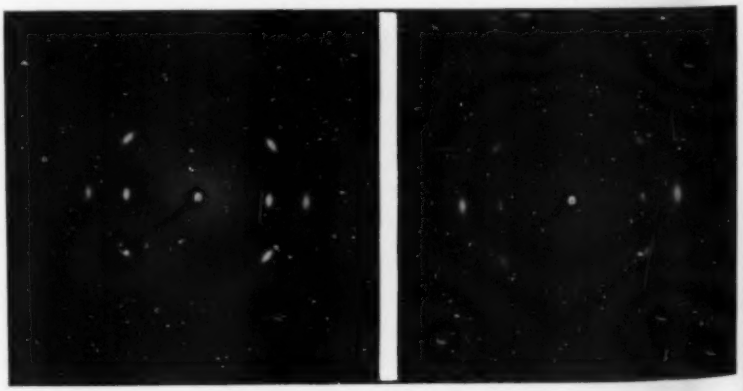


FIG. 16. Evaporated latex. Elongation: 900 percent, room temperature. Length: 2 inches; width: 3 inches; gauge: 0.039 inch.

FIG. 17. Evaporated latex. Elongation: 900 percent, room temperature. Length: 3 inches; width: 3 inches; gauge: 0.039 inch.

from the plotted values. There is no unanimity as to the letters used to designate the axes of the unit cell beyond the use of b for the fiber axis. Here, by the c axis is meant the axis with the dimension 12.6A.

Calculation of the percent of the diffracting crystallites with c axis in the plane of the sheet was made as follows:

Let N_c = number of crystallites with c axis in the plane of the sheet.

N_a = number of crystallites with a axis in the plane of the sheet.

It is assumed that the ratio N_a/N_c is proportional to the density ratio of the A_2 to the A_1 spots. It should be recognized that this assumption is true only if the crystallites have the same diffracting power under different conditions of stretching. When $N_a = N_c$, that is, for ordinary stretching, the ratio of the density of A_1 to A_2 was measured as 0.618. Use of this constant gives the formula

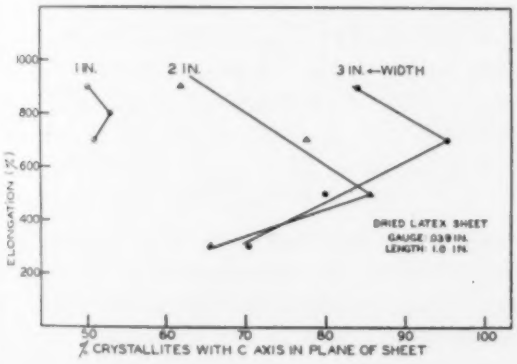


FIG. 18. Dependence of higher orientation on elongation

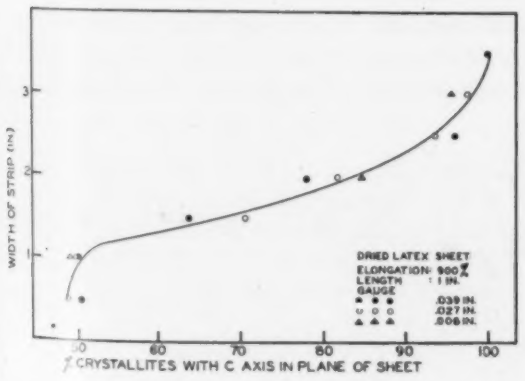


FIG. 19. Effect of width on higher orientation.

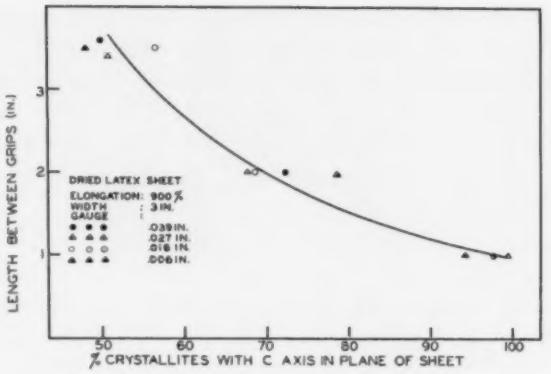


FIG. 20. Effect of length on higher orientation.

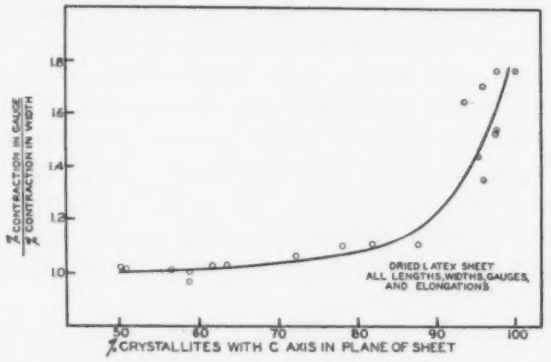


FIG. 21. Relation of higher orientation to dimensional changes during stretching.

Percent of diffracting crystallites with c axis in plane of sheet =

$$\frac{100N_c}{N_a+N_c}=\frac{100}{N_a/N_c+1}=\frac{100}{0.618D_2/D_1+1},$$

where D_2 and D_1 are the measured photographic densities

$$\left(D=\log_{10}\frac{I_{\text{background}}}{I_{\text{spot}}}\right)$$

for the A_2 and A_1 spots, respectively.

Figure 18 shows the effect of elongation and width of specimen, the gauge and length between grips being constant. In all the graphs, the gauge, width and length refer to original dimensions. The higher orientation reaches a maximum value as a function of the elongation for all the widths tried. This can be attributed to the fact that at the higher elongations the grips have moved so

far apart that they no longer serve to maintain the width of the stretched rubber. Consequently, the stretching process approaches that for a narrow piece.

In Fig. 19, the variables are the gauge and width, the elongation and distance between grips being constant.

Figure 20 illustrates the relationship between the length and the higher orientation for all gauges tried, the elongation and width being constant.

In Fig. 21 is plotted data for all gauges, widths, lengths between grips, and elongations shown in the previous graphs. By using as ordinate the ratio of the strain in gauge to the strain in width, the points fall on a smooth curve which rises rapidly for extreme values of higher orientation. Thus, however it is achieved, an increase in this ratio from unity seems to be the essential feature of a stretching process which gives rise to higher orientation. It was shown to be possible in some cases to improve the higher orientation by stretching a sample sidewise after

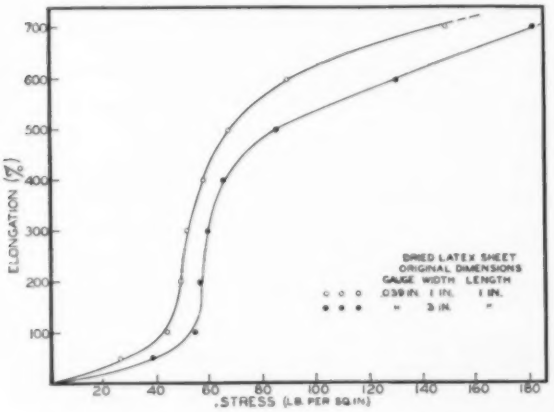


FIG. 22. Stress-strain curves.

it had been stretched lengthwise. If this is carried too far it results, of course, in random orientation.

Greater stress is required to stretch rubber with higher orientation than is required for ordinary stretching. This is shown by the curves of Fig. 22. This larger stress is associated with a more intense x-ray crystalline diagram. Figs. 23 and 24 illustrates this. They are comparable x-ray diffraction patterns at rather low elongations for ordinary stretching and for stretching with higher orientation, respectively. From the patterns, it is apparent that in the case of higher orientation the spots are stronger and probably would first appear at a lower elongation than is required in the case of ordinary stretching. The effect of maintaining the width during stretching may be considered as analogous to two-way stretching which, in turn, is equivalent to compression perpendicular to the plane of the sheet. The effect of pressure is to raise the fusion point of rubber.[15] Hence the conditions of stretching for higher orientation are such as to favor crystallization. In line with these considerations there is an edge effect which results in less perfect orientation near the edge of the sheet.

Figure 25 gives the same plot as Fig. 21 for several vulcanized pure gum stocks with different physical properties achieved by the use of different curing accelerators. The extent of the difference in physical properties can be seen from the data of Table I.

It is very difficult to discern any variation in the x-ray diffraction patterns for stocks as similar as *A* and *B*. The curves shown represent one means by which the different internal structures may be made apparent in the x-ray diffraction phenomena.

DISCUSSION OF RESULTS

The results of the stretching and freezing experiments can be discussed in the same terms as the crystallization of a supercooled liquid. The amorphous diagram, as a starting point, may be taken as evidence of a characteristic liquid structure for unstretched rubber. The molecular groups of the ordinary liquid are simulated by small portions of adjacent long chain molecules. The differences from an ordinary liquid are to be largely accounted for by the idea that a single

TABLE I.

COMPOUND	TENSILE STRENGTH LB./SQ. IN.	STRESS (500% ELONGATION) LB./SQ. IN.	FINAL ELONGATION %
A	1990	342	800
B	2110	427	730
C	3270	995	675

[15] P. A. Thiessen and W. Kirsch, Naturwiss. **26**, 387 (1938).

long chain molecule can extend, probably in zigzag fashion, through a large number of such groups or relatively ordered regions. These conceptions are, in a general way, in harmony with recent theories of micellar structure.[4–6]

Following the ideas of Tammann, the crystallization of a supercooled liquid proceeds from nuclear centers of crystallization formed in the liquid. These centers in rubber may be thought of as the ordered regions just discussed which undergo an improvement in internal molecular arrangement due to either a lowering of the temperature or stretching. The rate of formation of nuclei and the velocity of crystallization depend on a number of factors, important among which is the mobility of the molecules. The fact that rubber chilled suddenly to liquid-air temperature remains amorphous can thus be explained by the general theory of the production of the vitreous state.[16] The rate of temperature change, in this case, is large compared to the rate of formation of nuclei and the velocity of crystallization.

The size of the crystallites in stretched rubber, as determined from measurements of the size of the diffraction spots[2] remains practically unchanged as stretching proceeds. Hence the increased intensity of the spots is attributed to an increased number of diffracting regions. An improved lattice structure at higher elongations may also contribute to the greater intensity. The size of the crystals reached when a liquid crystallizes is determined by the relative rate of formation of nuclei and the velocity of crystallization. In the case of rubber, the number of nuclei set up by the stretching depends, among other things, on the amount of the stretch. The growth from these centers is limited, probably in a statistical way, by the low mobility of the molecules, that is, an equilibrium condition is reached between the movement of portions of the long chain molecules into the lattice and thermal agitation.

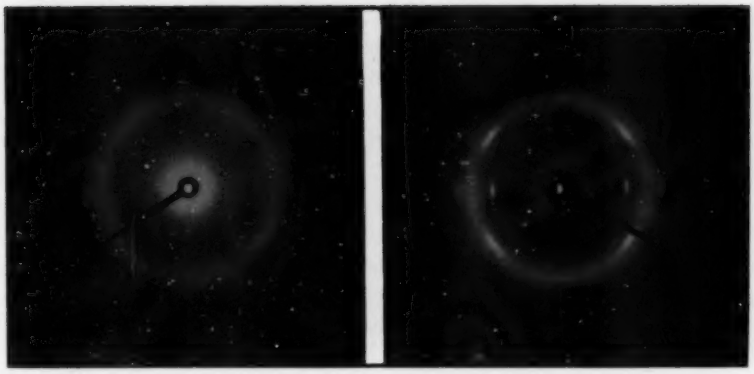


FIG. 23. Evaporated latex. Elongation: 300 percent, room temperature. Length: 1 inch; width: 1 inch; gauge: 0.039 inch.

FIG. 24. Evaporated latex. Elongation: 300 percent, room temperature. Length: 1 inch; width: 2 inches; gauge: 0.039 inch.

It remains to consider the combined effect of stretching and freezing. These experiments show that stretching gives rise to incipient nuclei of crystallization partially aligned with the axis of stretching, an idea not entirely new.[17] Crystal growth from these centers may be dependent upon the degree of perfection reached in the nuclei. Only at higher elongations at room temperature are the centers sufficiently perfect and definite to result in a crystal growth which becomes apparent by x-ray diffraction. When the temperature is lowered, centers, which may consist merely of an approximate parallelism of neighboring portions of a few long chain molecules, become effective for crystal growth. The velocity of crystallization from these centers is large compared to the rate of formation of new random centers due to the lower temperature. The result is a strong fiber diagram.

In the case of raw rubber, the perfection of the alignment of the nuclei along the axis of stretch and the number of nuclei is apparently inadequate to produce an appreciable fibering effect in the amorphous halo at room temperature before the diffraction spots appear. For vulcanized rubber such an effect has been reported.[18]

[16] A. Goetz and S. S. Goetz, J. App. Phys. **9**, 718 (1938).

[17] O. Gerngrosz, K. Herrmann and R. Lindemann, Koll. Zeits. **60**, 276 (1932).

[18] M. Iguchi and F. Schoszberger, Kautschuk **12**, 193 (1936).

The fact that stretched rubber can be frozen with higher orientation can be taken as an indication that the incipient centers of crystallization due to stretching possess in a rudimentary way all the symmetry characteristics of the crystal lattice.

The freezing of stretched raw rubber to give an oriented diagram is closely related to the phenomena of spontaneous molecular rearrangement observed for rubber by Thiessen and Wittstadt.[19, 20] By means of optical refraction and x-ray diffraction, they showed that orientation phenomena continued for a considerable period of time after the stretching. A definite equilibrium of the crystallized and amorphous components was reached, depending upon the temperature and other variables. Spontaneous volume changes after stretching have been observed for vulcanized rubber by Holt and McPherson.[21]

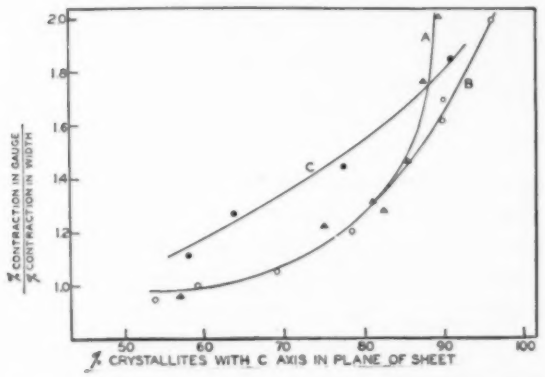


FIG. 25. Higher orientation for vulcanized rubber stocks.

The work of Thiessen and Wittstadt has led them to regard rubber as consisting of a polyphase system in which, within a wide range of temperatures, crystals and fused substance of varying compositions are in an equilibrium which is dependent upon the pressure. This conception of the crystallization process appears to be irreconcilable with a structure consisting of crystallites which are composed of mutually oriented portions of long chain molecules.

The most significant result from the study of the geometrical factors affecting the occurrence of higher orientation, from the standpoint of the internal structure of rubber, seems to be the quantitative connection between the extent of the higher orientation and the changes in lateral dimensions. Higher orientation results when the contraction in width of the stretched specimen is less than the contraction in gauge. This observation may have some connection with the shape of the unit cell,[12–14] the long dimension of which, in the case of higher orientation, lies in the direction of the width of the stretched rubber.

[19] P. A. Thiessen and W. Wittstadt, Zeits. f. physik. Chemie **B29**, 359 (1935).
[20] P. A. Thiessen and W. Wittstadt, Zeits. f. physik. Chemie **B41**, 33 (1938).
[21] W. L. Holt and A. T. McPherson, Nat. Bur. Stand. J. Research **17**, 657 (1936).

The Effects of Irradiation, Humidity and Sphere Material on the Sparkover Voltage of the Two-Centimeter Sphere Gap

ARTHUR B. LEWIS
University of Mississippi, University, Mississippi
(Received February 9, 1939)

Data have been obtained on the sparkover voltage of the two-centimeter sphere gap set at a gap of 4 mm (10.3-kv peak) showing that: (1) The effect of irradiation is to reduce the sparkover voltage by approximately 2.5 percent from its unirradiated value and to decrease the scattering of individual observations by a factor of about four. This irradiation effect is readily saturated by an open, cored-carbon, arc at 50 cm from the gap. (2) The effect of humidity, which is apparently independent of the sphere material for the five metals used here, is to increase the sparkover voltage by $+0.1_3$ percent per mm (of mercury) increase in vapor pressure of the water in the atmosphere. (3) There seems to be no choice between the metals used for spheres (aluminum, brass, chromium, nickel and steel) so far as repeatability of results is concerned, the probable error of a day's results averaging $\pm 0.2_8$ percent. This probable error can be largely if not wholly accounted for in terms of known sources of uncertainty. (4) The final sparkover voltages for the various metals, even when corrected to the same humidity, differ from each other by far more than can be accounted for by any definitely recognized source of uncertainty.

I. INTRODUCTION

THE sphere gap has in recent years been the subject of numerous publications[1] both because of the many interesting theoretical questions presented by the problem of the mechanism of the spark discharge and because of the great practical utility of the sphere spark gap as a high voltage measuring instrument. It is with the latter or practical aspect of the sphere-gap voltmeter that the present investigation is primarily concerned. The immediate purpose of this paper is to present data showing the effects of irradiation, humidity and sphere material on the sparkover voltage between two-centimeter spheres set 4 mm apart.

Some of these effects have been recognized by others.[2] In particular, quantitative work done in the High Voltage Laboratory of the National Bureau of Standards[3] has led to the following conclusions with respect to the 12.5-centimeter sphere-gap voltmeter operating in the voltage range between 30 and 100 kv. First, the effect of irradiation is to decrease the sparking voltage of a sphere gap (as compared with an unirradiated gap) by an amount which varies from two to five percent with different gap settings, and to decrease the scattering of successive observations of sparkover voltage by a factor of about five. Second, increasing the moisture content of the air causes an increase in the sparking voltage of a given gap which may amount to three percent under extreme conditions. Third, even under the most favorable circumstances there remains outstanding an uncertainty of the order of ± 0.25 percent when the 12.5-centimeter sphere-gap is used to measure voltage—an uncertainty which cannot be accounted for in terms of any of the known variables. It is hoped that, by extending these observations downward to the 10–30-kv range covered by the two-centimeter sphere gap, these several variables may be more readily separated and perhaps this unknown variable more easily identified under the simpler operating conditions existing at these lower voltages.

II. EXPERIMENTAL SET-UP

The sphere gap used followed closely the dimensions recommended in A. I. E. E. Standards No. 4 with respect to shank size, nearness of neighboring objects, etc. Five pairs of spheres were available; aluminum, brass, chromium (plated on brass), nickel (plated on steel) and steel. The aluminum, brass and chromium pairs were shaped to within about 0.1 percent of true size and true roundness (mean diameter = 20.00

[1] F. W. Peek, Jr., *Dielectric Phenomena in High Voltage Engineering* (McGraw-Hill, 1920); L. B. Loeb, Rev. Mod. Phys. **8**, 293 (1936).
[2] A. Vereecken, Bull. Soc. Belge Elec. **53**, 276 (1937); R. van Cauwenberghe and R. Marchal, Rev. Gen. d'Elec. **27**, 331 (1930); R. van Cauwenberghe, Soc. Franç. Elect. Bull. **7**, 1005 (1937).
[3] Unpublished results obtained by F. M. Defandorf, A. B. Lewis and A. W. Spinks in the High Voltage Laboratory of the National Bureau of Standards in 1935.

±0.02 mm) by the methods followed in grinding bearing balls. The steel and nickel plated pairs were ¾″ steel bearing balls which were closely round but were about five percent smaller than the desired 20 mm. The measured diameters of the spheres are given in Table I together with the voltage correction, if any, arising from the departure of the sphere diameters from exactly 20 mm. The origin of these voltage corrections will be discussed later under *Experimental Procedure.*

Alternating voltage was supplied to the spheres, one of which was grounded, by a separately excited motor-generator set and a step-up transformer. A 20,000-ohm resistor was placed in series with the high voltage lead to the sphere gap. No attempt was made at this time to investigate the effect on the sparkover voltage which might be produced by varying this series resistor. Voltage determinations were made by means of a voltmeter connected to the low voltage terminals of a step-down transformer whose high voltage winding was placed in parallel with that of the step-up transformer. Neither the ratio of transformation of the measuring transformer, nor the wave form of the high voltage wave supplied to the spheres, nor the voltage corrections to be applied to the voltmeter readings are known. It is not possible, therefore to interpret the voltmeter readings at the instant of sparkover in terms of crest volts across the gap. However, for approximately the same voltmeter readings, and for the same frequency and load, these factors should be constant, and the relative sparkover voltages in voltmeter-scale units should be strictly comparable. Unfortunately the behavior of both generator and transformers with respect to load changes and frequency changes leaves much to be desired.

TABLE I.

SPHERE MATERIAL	DIAMETER	VOLTAGE CORRECTION
Aluminum	20.00 mm	0.00%
Brass	20.03	0.00
Chromium	20.00	0.00
Nickel	19.08	+0.6₇
Steel	19.03	+0.7₄

Normal irradiation, except for those runs plotted in Fig. 1, was obtained with a 500-watt, cored-carbon, open arc operating on 110 volts, alternating voltage, and located 50 centimeters from the spheres. The spheres were protected from the direct heat of the arc and its ballast resistor.

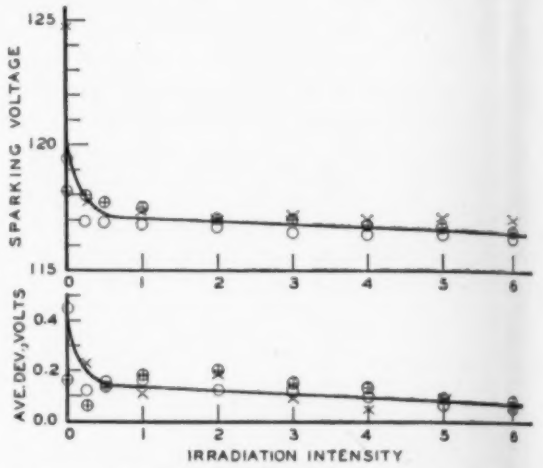


FIG. 1. The effect of varying intensity of irradiation on the sparking voltage and the scattering of observations of the two-centimeter sphere spark-gap set at a separation of 4 mm (10.3-kv peak). Both voltage and irradiation are measured in arbitrary units. Open circles refer to brass spheres, barred circles refer to aluminum spheres, and crosses refer to steel spheres.

As no suitable frequency meter was available it was necessary to determine the frequency of the supply voltage by reading the revolutions per minute of the generator shaft with a tachometer. The resulting uncertainty in machine speed was estimated to amount to at least ±10 r.p.m. It was experimentally determined that an increase in machine speed of 10 r.p.m., at 1800 r.p.m., caused an increase of 0.15 percent in the root-mean-square value of the voltage at which sparkover occurred. This observed change is taken as a measure of the changes in wave form and transformer ratio resulting from the change in frequency.

Sphere separations, room temperature and humidity were determined in the usual manner. The estimated uncertainties in these readings, and the resulting uncertainties in voltage determination, will be found recorded in Table II.

III. EXPERIMENTAL PROCEDURE

Before making a series of observations the spheres to be used were cleaned of all visible

oxide coating with No. 303 emery on chamois, washed with benzol and ethyl alcohol and dried, stem down. After adjusting the sphere separation to 4 mm, 10 preliminary or clean-up observations of sparkover voltage were made but discarded.

TABLE II. *Random errors affecting the voltage determinations.*

TYPE OF MEASUREMENT	MAGNITUDE OF UNCERTAINTY	RESULTING ERROR IN VOLTAGE IN PERCENT
Barometric pressure	±0.1 mm	$\pm 0.01_3$
Room temperature	±0.1°C	$\pm 0.03_2$
Vapor pressure of water present	±0.2 mm	$\pm 0.02_6$
Gap setting	$\pm 0.001_4$ mm	$\pm 0.03_2$
Machine speed	±10 r. p. m.	±0.15
	Total resulting random error	±0.16%

Ten consecutive determinations of sparkover voltage were then made and recorded. The average of these ten readings constitutes what will be called one run. The approach to sparkover voltage was made at such a speed that for the last four or five seconds preceding sparkover the voltage was increasing at the rate of approximately 0.5 percent per second.

The average voltmeter reading for each run was corrected to standard temperature and pressure in the usual manner (see A. I. E. E. Standards No. 4). The results given by the nickel and steel spheres were then corrected to equivalent voltages on 20.00-mm spheres according to Peek's formula[4] (see Table I).

This voltage correction arising from sphere diameter has been subjected to a partial experimental check. In the central curve of Fig. 3, labeled Ni, the open circles represent sparkover voltages taken with 19-millimeter spheres and corrected to the equivalent values on 20-millimeter spheres with the aid of the correction factor given in Table I. The solid circle plotted at 21 mm vapor pressure represents the average of 11 pairs of runs taken under identical experimental conditions except that 20-millimeter spheres were used. The spheres used to obtain these latter runs were later chromium plated. These data are taken to indicate the essential accuracy of the correction used. The status of this correction is, nevertheless, not very satisfactory. Curves of sparking voltage given by Weicker and Horcher and by Ver Planck[5] indicate that for the sphere sizes and gap used here the change in sparking voltage with sphere diameter may be substantially less than that indicated above.

[4] F. W. Peek, Jr., *Dielectric Phenomena in High Voltage Engineering* (1920), pp. 26, 27.

IV. RESULTS

1. Irradiation

Results showing the effect of irradiation with a cored-carbon arc on the sparkover voltage of aluminum, brass and steel spheres will be found plotted in the upper part of Fig. 1. The ordinates are sparkover voltages corrected for all known variables that affect the sparkover voltage except irradiation. The abscissas are irradiation intensities plotted in arbitrary units. The intensity of irradiation was varied by varying the distance between the arc and the sphere gap. In the lower section of the same figure will be found plotted the average deviation, expressed in volts, of an individual observation from the average of each corresponding group of ten observations constituting one run. These deviations have been plotted for each of the conditions of irradiation used. In the lower curve the average deviation for steel at zero irradiation was too large to be included on the figure as drawn. The solid lines in Fig. 1 were drawn in largely by eye through the average of the three sets of data. No attempt to discriminate between the various metals seemed advisable on the basis of these limited data. The zero irradiation, or "dark," point was weighted largely in favor of the brass spheres because of a large number of observations taken previously on these spheres. No great effort was taken to establish this dark point with certainty.

Three points are to be noted in connection with Fig. 1.

(1) The effect of irradiation on these spheres at this gap is to diminish the sparkover voltage by about 2.5 percent as compared with the unirradiated values.

(2) The effect of irradiation on these spheres at this gap is to diminish the average scattering of the individual observations by a factor of

[5] W. Weicker and W. Horcher, E.T.Z. **59**, 1029 and 1064 (1938); D. W. Ver Planck, Elect. Engineering **57**, 45 (1938).

about three or four as compared with the unirradiated values.

(3) Both these effects are seen to be essentially saturated at what has been called unit irradiation intensity. A sixfold increase of irradiation intensity beyond this point produces no further significant decrease in either sparkover voltage or scattering of observations.

The intensity of irradiation corresponding to that produced by the carbon arc at 50 cm, unit intensity of Fig. 1. was chosen as the standard experimental condition for further work. This is a reasonable compromise between the conflicting necessities of keeping the intensity of irradiation high and at the same time keeping extraneous objects a safe distance from the gap. This does not imply that the carbon arc is the most desirable source of irradiation, but merely the most convenient and economical one available.

2. *Humidity*

The effect of humidity on the sparkover voltage of this gap is shown by the data plotted in

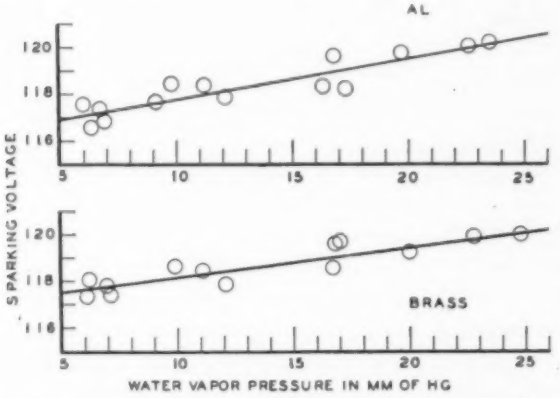


FIG. 2. The effect of humidity on the sparking voltage of the two-centimeter sphere spark-gap (aluminum and brass spheres, respectively) set at a four-mm gap (10.3-kv peak). Sparking voltage is expressed in voltmeter readings corrected for all known variables affecting the sphere gap indications except humidity. Humidity is expressed in terms of the water vapor present measured in mm of mercury.

Figs. 2 and 3. The ordinates are voltmeter readings at sparkover corrected for all known variables affecting sparkover except humidity. Each plotted point is the average of two runs of ten observations each. The abscissas represent the vapor pressure of the water in the atmosphere expressed in mm of mercury. The range of vapor pressures plotted in these figures represents the normal yearly range of humidity in the open laboratory in this locality.

The lines drawn in Figs. 2 and 3 are root-mean-square straight lines computed from the observed

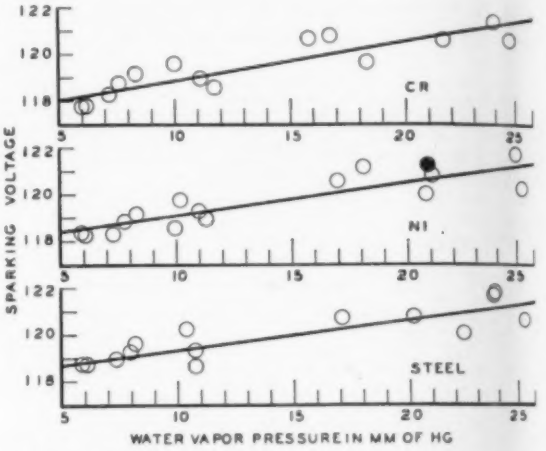


FIG. 3. The effect of humidity on the sparking voltage of the two-centimeter sphere spark-gap (chromium, nickel and steel spheres, respectively) set at a four-mm gap (10.3 kv-peak). Sparking voltage is expressed in voltmeter readings corrected for all known variables affecting the sphere gap indications except humidity. Humidity is expressed in terms of water vapor present measured in mm of mercury.

points. There is some indication, as has been previously observed,[3] that the sparkover voltage tends to increase more slowly at the higher vapor pressures and to decrease more rapidly at the lower vapor pressures. The present data, however, hardly justify more than a straight line.

V. DISCUSSION

It is obvious from an inspection of Figs. 2 and 3 that it is necessary to specify the vapor pressure of the water present if the sparkover voltage of this gap is to be specified accurately. The vapor pressure chosen as a standard, to which all voltages will be corrected, will be 16 mm of mercury. This is close to the average vapor pressure observed in this locality during most of the year and is also close to the standard conditions recommended in A. I. E. E. Standards No. 41 for Insulator Tests.

These sparkover voltages at 16 mm vapor pressure together with the slopes of the lines in Figs. 2 and 3, expressed as percent increase in

voltage per mm increase in vapor pressure (both data taken from the equations of the root-mean-square lines) will be found in columns 2 and 3 of Table III.

It is believed that the differences in slopes recorded in column 3, Table III, are not significant. We conclude, then, that for the five metals used here the effect of humidity on the sparkover voltage of the irradiated two-centimeter sphere gap, at a four-mm gap, is independent of the sphere material and amounts to an increase in sparkover voltage of $+0.1_3$ percent per mm increase in water vapor pressure. This value of slope is in good agreement with that previously observed when working with the 12.5-centimeter sphere gap,[3] the value of slope in this latter case being $+0.1_4$ percent increase in voltage per mm increase in vapor pressure over the same pressure range.

We now inquire as to the probable errors of these voltage measurements made with the irradiated sphere gap. In column 4, Table III, will be found the probable errors, expressed in percent, of the individual points plotted in Figs. 2 and 3. If all the sources of uncertainty affecting a given day's runs are known and accounted for, it should be possible to account numerically for these probable errors. Now the points plotted in Figs. 2 and 3 each represent the average of two runs. These runs, although always taken consecutively on the same day, always differed from each other and the plotted points are therefore uncertain by a corresponding amount. The most probable value for these observed daily differences is ± 0.14 percent, there being no significant differences between the various metals used for spheres. In addition the final corrected voltages observed on any given day are uncertain because of uncertainties in observing the barometric pressure, room temperature, machine speed, etc. This uncertainty amounts to ± 0.16 percent (see Table II). These two sources of uncertainty can therefore account for an uncertainty in the plotted points of Figs. 2 and 3 of ± 0.21 percent. This is of the same order of magnitude as the experimentally observed probable errors recorded in Table III. We therefore conclude that there are no unrecognized sources of uncertainty of magnitude greater than ± 0.2 percent affecting the observed voltages.

Now consider the corrected voltages for the various metals recorded in column 2, Table III, as V_{16}. Each of these voltages is the average of at least 14 sets of observations. From the magnitude of the probable errors recorded in column 4, Table III, we would expect the probable errors of these means (V_{16}) to be of the order of magnitude of ± 0.10 percent. It is obvious that the sparkover voltages for the different metals differ from each other by far more than this amount. These data therefore indicate that the sparkover voltage of this irradiated gap depends on the sphere material.

TABLE III. *Summary of results.*

SPHERES	SPARKOVER VOLTS, V_{16}	SLOPE IN PERCENT OF V_{16} PER MM	PROBABLE ERROR IN PERCENT OF V_{16}
Aluminum	118.8	+0.15%	±0.27%
Brass	119.0	+0.11	±0.23
Chromium	119.9	+0.14	±0.31
Nickel	119.9	+0.12	±0.30
Steel	120.1	+0.11	±0.31
Average	119.5	+0.13	

VI. ACKNOWLEDGMENTS

The author is indebted to Drs. F. B. Silsbee and F. M. Defandorf, both of the National Bureau of Standards, for assistance and many helpful discussions during the course of this investigation and during the preparation of this paper.

The Thermal Distribution and Temperature Gradient in the Arc Welding of Oil Well Casing

W. A. Bruce
Carter Oil Company, Tulsa, Oklahoma
(Received January 26, 1939)

The problem studied here is the thermal distribution due to a number of moving heat sources. It is assumed that the pipe is a circular cylinder and that the welders move on a circle of the cylinder. The Kelvin method for determining the thermal distribution about a stationary source of heat is extended to a case in which the source of heat is moving with a constant velocity. By integration of the effect of stationary sources the general effect of a moving welder is obtained. It is shown that an infinite plate with an infinite number of sources on a line and w cm apart is equivalent to an infinite cylindrical shell with $n=c/w$ sources spaced w cm apart around the cylinder (c=circumference). With the aid of this and Kelvin's solution it is possible to build a complete general solution of the problem. This method of constructing the solution of a heat problem has many practical applications—such as the determination of thermal distributions in the welding of plates, seamed pipe and other bounded objects.

Introduction

OIL well casing is usually supplied in 40-ft. sections. These sections are either screwed or welded together at the well. Various techniques have been employed for welding these sections together. The usual practice is to have 2 or 3 welders start simultaneously at equal intervals around the pipe and proceed until each man comes to the point where the next man started. A weld is completed with 2, 3 or 4 layers, or beads, and after a minute or so of cooling it is lowered into the hole.

There are many advantages in this welded joint but one disadvantage is the failure of a joint now and then. Examinations of these failures show that in almost every case the welded joint exhibits microscopic cracks and an excessive hardness. Quenching tests indicate that these cracks appear most frequently when the weld is cooled with water before a certain time has elapsed. Most failures are eliminated when the weld is allowed to cool in air.

It was known that a complete metallurgical understanding of this problem and an exact knowledge of this critical time of quenching could not be achieved unless we knew exactly the temperatures *vs.* time and rates of cooling at the weld; hence, this problem was attacked.

The following is merely a solution of the thermal problem; however, certain experimental measurements were made to verify this mathematical development. The nature of these experiments is not discussed here but very good agreement was obtained between analytic and experimental results.

The Heat Equation

The equation for the flow of heat where there is radiation from the surface is given by a number of authors.[1–4] It is

$$k\nabla^2\theta_0=\partial\theta_0/\partial t+\lambda\theta_0, \tag{1}$$

where k=diffusivity, θ_0=temperature, t=time, and λ=a radiation constant. If θ_0 is expressed by

$$\theta_0=\theta\cdot e^{-\lambda t} \tag{2}$$

Eq. (1) becomes

$$k\nabla^2\theta=\partial\theta/\partial t. \tag{3}$$

Methods for finding a function which will satisfy (3) are discussed by Forsyth,[5] Whittaker and Watson,[6] and Bateman.[7]

[1] F. S. Woods, *Advanced Calculus* (Ginn and Company Boston, 1932).
[2] H. S. Carslaw, *The Conduction of Heat* (MacMillan and Company, Ltd., New York City).
[3] W. E. Byerly, *Fourier Series and Spherical Harmonics* (Ginn and Company, Boston, 1893).
[4] *The Papers of Lord Kelvin;* see Lord Rayleigh, Phil. Mag. **22**, 381 (1911).
[5] A. R. Forsyth, *Theory of Differential Equations* (Cambridge University Press, 1890–1906), Vol. V, Part 2 and Vol. VI, Part 4.
[6] E. T. Whittaker and G. N. Watson, *Modern Analysis* (Cambridge University Press, London, 1935).
[7] H. Bateman, *Partial Differential Equations* (Cambridge (MacMillan and Company, New York)).

If the discussion is limited to two dimensions it can be shown by substitution that

$$\theta=\int\int_{-\infty}^{\infty}\frac{f(x',y')}{4\pi kt}\cdot e^{-[(x-x')^2+(y-y')^2]/4kt}\cdot dx'dy' \quad (4)$$

is a solution of Eq. (3).

This is the Fourier solution applied to an infinite flat plate. It is most useful in heat flow and other types of flow problems because of its properties of vanishing for infinite values of x' and y' and of becoming equal to an arbitrary function, $f(x', y')$, at $t=0$.

A Source of Heat

Kelvin[4] was responsible for the method of heat sources that will be used here. In order to consider a line source of heat in a flat metal plate of diffusivity, k, and thickness, b, it is necessary to imagine a small rectangular parallelepiped of metal, abh, placed inside the plate and extending from $x=-a/2$ to $x=a/2$ and $y=-h/2$ to $h/2$. The parallelepiped is at temperature, V. Heat will start to flow out of this small source at $t=0$, the instant it is placed in the plate, and it will flow in accordance with Eq. (4), the solution of the heat equation. At $t=0, f(x', y')=0$ everywhere, except inside the block, abh, where $f(x', y')=V$.

Thus

$$\theta=\frac{V}{4\pi kt}\cdot\int_{-h/2}^{h/2}\int_{-a/2}^{a/2}e^{-[(x-x')^2+(y-y')^2]/4kt}\cdot dx'dy'. \quad (5)$$

For the moment consider a welder working upon an infinite plate. It is possible that he can deposit a hot drop of metal—say, 0.5 cm wide, 0.01 cm thick, and b cm deep—in a fraction of a second, δt. Thus, if we consider that he has, during time δt, deposited the above rectangular piece of metal we see that the integral (5) will become, to a first-order approximation,

$$\theta=\frac{Vha}{4\pi kt}\cdot e^{-(x^2+y^2)/4kt}. \quad (6)$$

The mathematical analysis of the heat distribution due to a moving welder has been made by a number of authors.[8–10] Rosenthal,[8] and Lance-Martin and Boulton[9] have used the Kelvin method of sources and have arrived at the same expression for one welder moving in an infinite plane. Portevin and Seferian[10] have used the Fourier method.

For the present we consider Eq. (6) suitable but remember that it becomes in error for small values of the time.

If the welder is to operate at some place other than the origin (say, x' and y') the necessary changes in variables causes (6) to become

$$\theta=\frac{Vha}{4\pi kt}\cdot e^{-[(x-x')^2+(y-y')^2]/4kt}. \quad (7)$$

If a source were left by a welder who was leaving heat at the rate q calories per second and simultaneously moving in the x direction along the x axis at the rate $dx'/dt=v$, the relation between q and V would be given by

$$\begin{aligned} &qdt=V\delta cbhdx',\\ &\delta=\text{density of metal plate},\\ &c=\text{specific heat}, \end{aligned} \quad (8)$$

where now $a=dx'$. Then

$$q/vc\delta b=Vh \quad (9)$$

and

$$\theta=\frac{q}{4\pi kvc\delta bt}\cdot e^{-[(x-x')^2+(y-y')]/24kt}\cdot dx'. \quad (10)$$

Eq. (10) represents the thermal distribution due to the quantity of heat deposited in the time dx'/v. This heat will distribute itself throughout the infinite plane in accord with (10) and at time t later the isotherms will be concentric circles whose centers are x', y'.

The Use of Image Sources

In connection with the well-known problem of the Fourier ring it has been shown[2] that a single source on a thin ring of circumference c cm is

[8] D. Rosenthal, *A Theoretical Study of The Temperatures in The Region of The Arc* (*National Congress of Science*, Brussels, 1935). Also, Welding Journal, April, 1938.

[9] N. S. Boulton and H. E. Lance-Martin, "Theoretical Study of The Temperature about an Arc," Proc. Eng. Soc. (British).

[10] A. Portevin and D. Seferian, "The Temperature Distribution about a Moving Arc," The Welding Journal, September, 1937. (Chaleur et Ind. **16**, 409 (1935)).

equivalent to an infinite number of such sources placed at intervals of c cm along a straight line. This is true because in either case $\partial\theta/\partial x=0$ at the point $c/2$ from a source.

This idea may be extended to the consideration of a plane. If we have an infinite number of sources spaced w cm apart along the x axis in a plane, Fig. 1, it can be shown that $\partial\theta/\partial x=0$ for $x=x'+(n-\frac{1}{2})w$. Likewise, if we have a source of heat on a thin circular cylinder of circumference w cm, no heat will flow across the element of this cylinder which is on the opposite side, or $w/2$ cm, from the source. Thus, as in the case of the ring and the line, the heat problems of the infinite sources along a line in a plane and of the source on a thin cylinder are the same. Furthermore, if we have n sources spaced equally on a circle of a cylinder the equivalence of the plane and the cylinder problems is still apparent if the separation of the sources in the plane is $w=c/n$.

The Moving Arc

We return to the infinite plane with the sources w cm apart. The temperature at some point x, y might be computed by consideration of the bound slab $-w/2$ to $+w/2$ but it is easier to consider

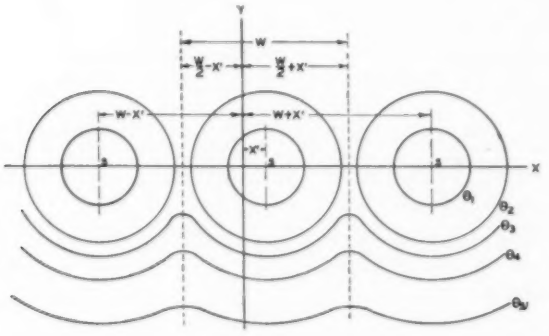


FIG. 1. Picture of the isotherms at some time after the image sources are placed at the points s in the flat plate. This picture is repeated from $x=-\infty$ to $+\infty$, $y=0$.

the virtual sources at the positions $x'+nw$, $y'=0$ where n has all integral values from $-\infty$ to $+\infty$. Consider the effect of the center source (see Fig. 2) which at some time t' is at x' and is moving toward 0 with a velocity v. As it moves a distance dx' it sends out heat which will arrive at x, y, t seconds later to produce a temperature given by Eq. (7). If the source travels with a constant velocity v it will arrive at 0 in $t=x'/v$ seconds and if we start counting time from the time the source arrives at 0 then T seconds later we have $t=t+T=x'/v+T$. We may sum the effect of its course from $+w$ to 0 by integrating Eq. (10) over the path w to 0 (t becomes $x'/v+T$).

$$\theta=\frac{q}{vc\delta b}\int_0^w \frac{1}{4\pi k((x'/v)+T)} \cdot e^{-[(x-x')^2+(y-y')^2]/4k(x'/v+T)}\cdot dx'. \quad (11)$$

In considering the infinite sources of the plane we may assign index numbers for them; say $n=0$ for the one which finishes at $x=0$, $n=1$ for the one finishing at $x=w$, etc. As source 0 is going from w to 0 source 1 is going from $2w$ to w and source 2 from $3w$ to $2w$, etc. The point of observation remains the same and only the variable of the source changes. Proper allowance for the change in variable is shown in Eq. (12) where n is the index number of any virtual source. Therefore it follows that that part of the temperature which is due to the nth source is given by the expression,

$$\theta_n=\frac{q}{vKb4\pi}\int_{nw}^{(n+1)w} \frac{1}{(x'/v+T)} \cdot e^{-[(x-x'-nw)^2+y^2]/4k(x'/v+T)}\cdot dx', \quad (12)$$

where $K/\delta c=k$. Then the total effect of all the virtual sources is the sum of the expressions (12) where the index number ranges from $-\infty$ to $+\infty$. It is necessary to include here the variable change of Eq. (2). This is done and the subscript of θ is dropped. Thus the temperature at any given point x, y and at any time T after the welders break arc is given by

$$\theta=\sum_{n=-\infty}^{n=+\infty}\frac{q}{4\pi Kvb}\int_{nw}^{(n+1)w} \frac{1}{(x'/v+T)}\cdot e^{-\lambda(x'/v+T)} \cdot e^{-[(x-x'-nw)^2+y^2]/4k(x'/v+T)}\cdot dx'. \quad (13)$$

Cylindrical Coordinates

In dealing with casing which is, as a rule, circular tubing it is natural to think of it in terms of cylindrical coordinates. The problem is greatly simplified over the usual cylindrical coordinate

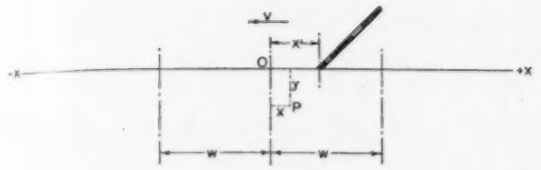


FIG. 2. An arc source of heat at $(x', 0)$ moving with a constant velocity, v.

problem in that for any specific casing we will assume that r is a constant and is the mean radius of the casing; thus, the angular coordinate may be replaced by a circumferential coordinate —that is, we may say the distance from any arbitrary point on the circumference to some point $r\phi$ is equal to x. Thus, in any specific case we may use the coordinates x and y where it is understood that x is the distance measured along the circumference, any y is the vertical distance. Because we have shown that the plane and cylinder may be interchanged, no ambiguity or confusion need arise from the use of the coordinates x and y when considering the cylindrical problem.

THE USE OF BESSEL FUNCTIONS

We may simplify Eq. (12) by making the following substitutions:

$$\begin{aligned} q' &= q/4\pi Kb, \\ \mu &= 1+2k\lambda/v^2, \\ \xi &= (v/2k)(x+vT-nw), \\ \xi &= \xi_0\mu^{\frac{1}{2}}, \\ s &= (v/2k)(x'+vT)\mu, \\ A &= (v/2k)(nw+vT)\mu, \\ B &= (v/2k)([n+1]w+vT)\mu. \end{aligned} \tag{14}$$

Thus,

$$\theta_n = q'\int_A^B e^{\xi}\cdot e^{-\frac{1}{2}(\xi_0^2/s+s)}\cdot\frac{ds}{s}. \tag{15}$$

It may be shown that the integral of (15) is related to the Bessel function of the second kind under certain conditions which are observed in practical casing welding. The formal relation is[11]

$$K_0(\xi) = \tfrac{1}{2}\int_0^{\infty} e^{-\frac{1}{2}(\xi^2/s+s)}\cdot\frac{ds}{s}. \tag{16}$$

[11] Andrew Gray, G. B. Mathews and T. M. McRobert, *A Treatise on Bessel Functions and Their Application to Physics* (MacMillan and Company, London, 1931).

APPROXIMATE FORMULAE

We start with Eq. (13) and consider the temperature at the point $x=0$, $y=0$ and assume that the radiation is negligible. As more welders are used to weld a given diameter pipe, w becomes less and the situation approaches that of a ring of heat being applied at the welding circle, $y=0$. When w is very small x' may be neglected and

$$\theta = \frac{q'}{c\delta b}\sum_{n=-\infty}^{n=\infty}\frac{w}{4\pi kvT}\cdot e^{-(nw)^2/4kT}. \tag{17}$$

Let w pass to the limit and write $w=d(nw)$. Then

$$\theta = \frac{q'}{c\delta b}\int_{-\infty}^{\infty}\frac{1}{4\pi kvT}\cdot e^{-(nw)^2/4kT}\cdot d(nw) \tag{18}$$

or

$$\theta = \frac{q'(4\pi k)^{\frac{1}{2}}}{v}\cdot\frac{1}{T^{\frac{1}{2}}}. \tag{19}$$

The approximate formula (19) may be obtained by an entirely different approach. Consider an infinite cylinder into which there has been introduced a ring of heat of density Q calories per sq. cm. The heat density is $Q=nqt/\pi bd$ where t is the welding time of the n welders and $t=\pi d/nv$; hence, $Q=q/vbc\delta$.

Then by the same process used in finding Eq. (7)

$$\theta = \frac{q}{vbc\delta}\cdot\frac{1}{(4\pi kT)^{\frac{1}{2}}}\cdot e^{-v^2/4kT}. \tag{20}$$

We see that Eq. (20) becomes equivalent to Eq. (19) when y is equal to 0—i.e., along the line of welding. Eq. (19) is taken as the value of θ for the limiting case of welders spaced infinitely close together. It is developed from Eq. (13), which in turn involves all of the assumptions and proofs necessary for the development of our solution (Eq. (13)). Eq. (20) is developed by considering one bead of the weld as being made by the instantaneous application of heat at all points around the weld. In a practical case the values of θ given by the general equation will approach the values obtained from the approximate equation in (19) or (20) or values of T of about 180 seconds. Obviously, if we are to con-

sider two or three welders as having produced an average amount of heat per unit area, the time involved should be the average time from when they produced the heat; thus, the time in Eq. (20) must be one-half the time required to make a weld plus the time elapsed after the arc has broken.

Rate of Cooling

The $\partial\theta/\partial t$ taken in Eq. (15) and summed for all values of n will give the rate of cooling. A good approximation for $\mu=1$ and $\xi_0<vw/k$ is

$$\partial\theta/\partial t=\partial\theta_0/\partial t+\partial\theta_{-1}/\partial t, \tag{21}$$

where

$$\partial\theta_0/\partial t=q'\int_A^B\left(1-\frac{\xi_0}{s}\right)\cdot e^{\xi_0}\cdot e^{-\frac{1}{2}(\xi_0^2/s+s)}\cdot\frac{\partial\xi}{\partial t}\cdot\frac{ds}{s}-\frac{q'}{\xi_0}\cdot\frac{\partial\xi}{\partial t},$$

$$\partial\theta_{-1}/\partial t=q'\int_A^B\left(1-\frac{\xi_{-1}}{s}\right)\cdot e^{\xi_{-1}}\cdot e^{-\frac{1}{2}(\xi_{-1}^2/s+s)}\cdot\frac{\partial\xi}{\partial t}\cdot\frac{ds}{s}+\frac{q'}{\xi_{-1}}\cdot\frac{\partial\xi}{\partial t}. \tag{22}$$

If $\xi_0>vw/k$ the partial of (19) gives

$$\frac{\partial\theta}{\partial t}=-q'-\frac{(4\pi k)^{\frac{1}{2}}}{2v}\cdot T^{-\frac{1}{2}}. \tag{23}$$

If Bessel functions can be used and $\mu=1$

$$\frac{\partial\theta_n}{\partial t}=e^{\xi}[K_1(\xi)-K_0(\xi)]\frac{\partial\xi}{\partial t}. \tag{24}$$

Figure 3 shows the rate of cooling for the conditions of the practical problem given below.

Other Welding Problems

The use of the method of images described above lends itself to the study of the thermal distribution in many welding problems. An investigation of flow problems in general will show that this method is most useful in the study of flow in bounded regions.

If cylindrical tubing is to be welded along an element of the cylinder and the tube is long in comparison to its diameter, as it is in practical cases

$$\theta=\sum_{n=-\infty}^{+\infty}2\cdot q'\cdot e^{x/u}\cdot K_0(\xi_n), \tag{25}$$

where $u=2k/v$ and $\xi_n=[x^2+(y-nl)^2]^{\frac{1}{2}}/u$. ($l$ is the circumference of the pipe.)

In this problem the axis is considered as moving with the welding arc—x is the distance along the length of the pipe measured in an opposite sense from the velocity of the arc, and y is the circumferential coordinate, Fig. 4(a). The other variables have been defined.

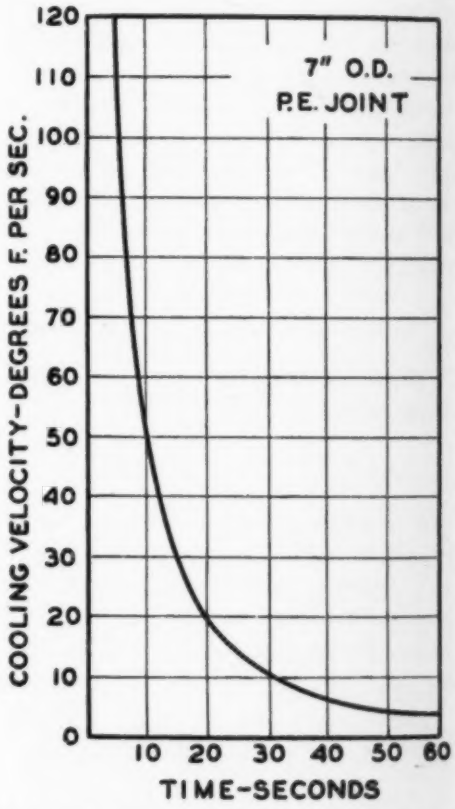


Fig. 3. Rate of cooling of weld on 7″×0.339″ casing.

In the case of narrow flat plates, Eq. (25) is useful. Here y is the distance from the line of welding to the point under observation and l is the width of the plates when joined, Fig. 4(b).

A great many heat problems in welding are similar to the problem of welding with a backing plate, Fig. 4(c). It can be shown that all of these can be treated as though an additional thickness

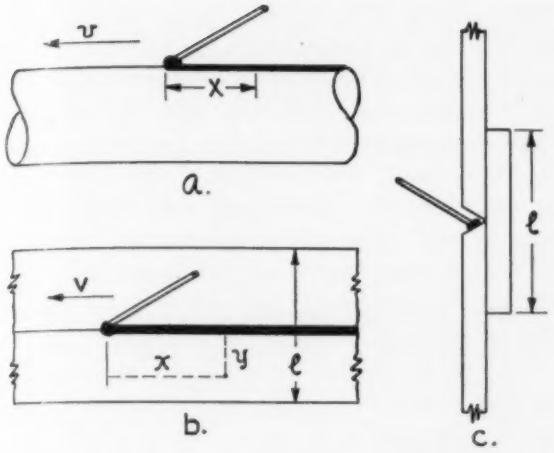


FIG. 4. (a) Longitudinal welding of pipe. (b) Welding flat plates. (c) Welding with backing plate.

were added to the original plate and that the added thickness appears to decrease as time increases. The factor by which the temperature must be multiplied to correct for a backing plate is

$$\frac{1}{1+(b'/b)(1+e^{-l^2/kt}+e^{-4l^2/kt}+\cdots)}, \quad (26)$$

where b is the thickness as defined before, b' is the thickness of the backing plate, and l is the half-width of this plate.

In the case of a bell and spigot pipe joint this correction factor will be

$$\frac{1}{1+0.44/(1+e^{-l^2/kt}+e^{-4l^2/kt}+\cdots)}, \quad (27)$$

where l is the length of the bell and it is assumed that the thickness of the bell is 0.88 times the thickness of the spigot.

A Practical Example of Casing Welding

The use of Eq. (13) to determine the cooling curve of the points where the welders break arc is a special case of the general solution. The welders seldom stand closer together than 6″ and their speed is usually about 6″ per minute. Practically, it has been found that under these conditions it is necessary to consider only the -1 and 0 virtual sources of Eq. (13) for values of T smaller than 150 seconds. Also it has been found that if the welders are as close together as 8″ (7″ casing, 3 welders) the approximate formula (19) may be used for T greater than 180

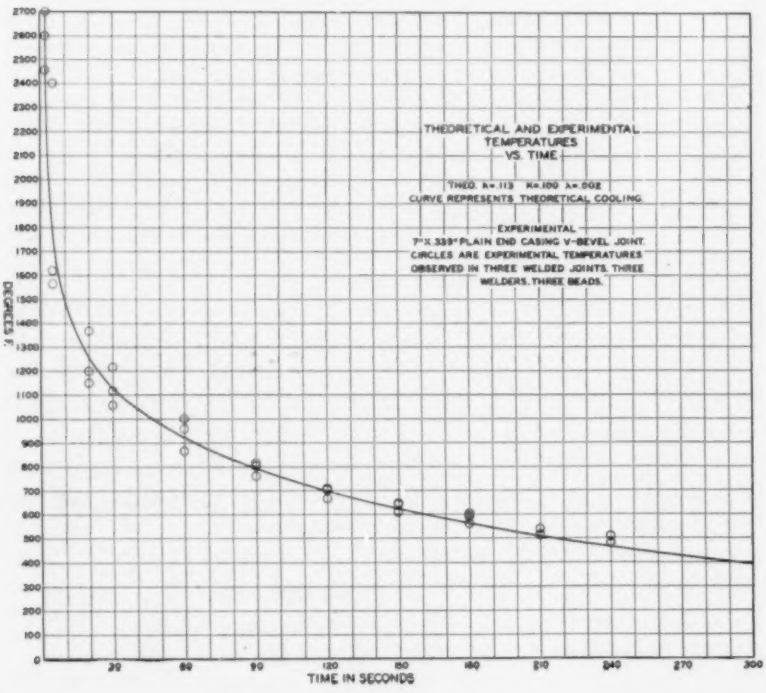

THEORETICAL AND EXPERIMENTAL TEMPERATURES VS. TIME
THEO. h=.113 K=.100 k=.502
CURVE REPRESENTS THEORETICAL COOLING
EXPERIMENTAL
7" X .339" PLAIN END CASING V-BEVEL JOINT.
CIRCLES ARE EXPERIMENTAL TEMPERATURES OBSERVED IN THREE WELDED JOINTS. THREE WELDERS. THREE BEADS.
DEGREES F.
TIME IN SECONDS

FIG. 5. Each experimental point is the average of three readings. The temperatures were measured by welding thermocouple wires into the last bead as the welders finished.

seconds. A typical example is as follows:

7″ O.D. × 0.339″	Conductivity = $K = 0.100$
3 welders	Radiation constant = $\lambda = 0.002$
3 beads	Speed of welding = $v = 0.250$
Diffusivity	cm/sec.
$= k = .113$	Plain end V-bevel casing

In this, $x = y = 0$.

Since λ is small, it is very good practice to eliminate the cooling factor from the integrand and to allow for the cooling effect by the method described above—i.e., $\theta_0 = \theta \cdot e^{-\lambda T}$.

To solve a large number of problems it was found easiest to plot the integrand of Eq. (15) for a number of arbitrary values of ξ. For any specific problem a curve of θ_n *vs.* ξ was drawn by integrating graphically the curves between the limits A and B and hence for θ *vs.* T, θ_1 and θ_0 were added and multiplied by $e^{-\lambda T}$ for the true temperature. After this process was carried out to 150 seconds, we used Eq. (19) and also the corresponding radiation factor. Fig. 5 is a typical result for the 7″ joint mentioned above. The constants used were experimentally determined. The rate of heat input of each welder was taken as 695 calories/second. In all this work it is possible to use the Bessel function tables[12] to a considerable advantage.

The author wishes to thank the Management of The Carter Oil Company for permitting the publication of this article.

[12] *British Association Mathematical Tables* (Bessel, Hankel and Associated Functions).

Convection and Conduction of Heat in Gases

I. Brody and F. Kőrösy
Tungsram Research Laboratory, Ujpest near Budapest, Hungary

(Received July 18, 1938)

The diameter of the "Langmuir film" (b) around warm filaments in gases was at first determined by Langmuir's own method in A, Kr and N_2. The calculated film thickness beside a plane surface is 3.3 mm, 1.4 mm and 4.3 mm, respectively. The said diameter was then redefined as the distance of thermal interaction between two similar, warm filaments; temperature measurements revealed that this conventional definition corresponds to the 90°C isothermals. b is a linear function of the temperature; $b_t = b_{1000}[1 + (t - 1000) \cdot 0{,}000255]$ and b_{1000} decreases with increasing molecular weight of the gases. b decreases with increasing gas pressure: $b/b_1 = (p_1/p)^{0.42}$ and it increases somewhat slower with the diameter of the filament (a) than would correspond to the equation of Langmuir: $2B = b \cdot \ln b/a$. The detailed temperature field around warm filaments was determined under varied conditions. Evidence was gathered, that although 90 percent of the wattage lost can be accounted for by assuming pure conduction, the gas is definitely moving upwards within the film as well as outside of it.

LANGMUIR'S fundamental work on heat losses of incandescent wires[1] revealed that practically the whole temperature drop around the wires is confined to a relatively small volume. Langmuir concluded that "the loss of heat from wires by free convection takes place exactly as if there were a film of stationary gas around the wire, through which the heat is carried entirely by conduction." It was of course evident, that this hypothesis could only be a crude approximation of the facts; the gas must needs rise in the neighborhood of the warm filament. The hydrodynamical treatment of the problem is very complicated and gave only dimensional equations of similarity, connecting the watt consumption with the variation of different parameters.[2] Our object was rather to examine the geometrics of

[1] I. Langmuir, Phys. Rev. **34**, [1], 401 (1912); Proc. A. I. E. E. **31**, 1011 (1912); Ch. W. Rice, J. A. I. E. E. **42**, 1288 (1923).

[2] A. Oberbeck, Ann. d. Physik **7**, 291 (1879); J. Boussinesq, Comptes rendus **132**, 1382 (1901); A. H. Davis, Phil. Mag. **40**, 692 (1920); **43**, 329 (1022); **44**, 920, 940 (1922); **47**, 1057 (1924); R. Hermann, "Wärmeübergang bei freier Strömung am waagerechten Zylinder in zweiatomigen Gasen," V. D. I. Forschungsheft No. 379 (1936); W. Elenbaas, Physica **4**, 761 (1937).

the film under conditions varying from the effect of these on the wattage. We therefore tried to gather more experimental facts concerning free convection around incandescent filaments and compared our results with the deductions from Langmuir's hypothesis.

Diameter of the Gas Film in A, Kr, N_2 by the Original Langmuir Method

First of all we extended the measurements by Langmuir's own method to the three gases, argon, krypton and nitrogen. We used a tungsten filament of 0.099 mm diameter, 227 mm long, extended in a horizontal glass tube of about five cm diameter by means of a molybdenum spring. Its temperature was calculated from the electric resistance (the current did not pass the molybdenum). The watt consumption was determined at known temperatures of the filament and from this value the watt consumption in vacuum at the same temperatures was subtracted. The differences gave the losses due to the gas around the filament. The pressure was always one atmos. (1.04M bar). *Langmuir assumes* that a practically

Table I. *Thickness of the "gas film" (b) calculated from experimental w data, at various temperatures. $a = 0.099$ mm. Horizontal filament.*

	b		
T_{abs}	A	Kr	N_2
1050	2.1 mm	1.2 mm	2.0 mm
1315	1.6	1.1	2.6
1537	2.1	0.7	2.5
1735	2.3	1.0	2.9
1910	2.5	1.5	3.0
2060	2.6	1.7	2.9
2235	3.7	1.7	3.3
2350	4.3	2.1	2.9

immobile, conductive, gas film around the filament extends to a boundary where the gas temperature is practically equal to the temperature of the gas vessel, say 400°K. *Assuming* pure conduction within this layer, the equation of heat conduction may be integrated for a filament with diameter a and gas layer with diameter b, having 400°K temperature at its borders:

$$\ln \frac{b}{a} = 2\pi \int_{400}^{T} K dT \Big/ w, \qquad (1)$$

where K is the heat conductivity of the gas and w the watt loss owing to the gas, per unit length of the filament. T is the temperature of the filament. The temperature dependence of K must be calculated on basis of the kinetic theory of gases from their specific heats and viscosities, using Sutherland's formula for the latter.[1] We thus calculated from our experimental w data the thickness of the hypothetical gas film and arrived at the values given in Table I. These values correspond to the given diameter of the filament. By *assuming* that not only the temperature (400°K), but also its gradient is independent from a at the border of the stationary layer, Langmuir calculated the thickness of the stationary layer ($=B$) beside a plain surface of the same temperature as the filament:

$$B = (b/2) \ln (b/a). \qquad (2)$$

Table II. *Thickness of the "gas film beside a vertical plane" calculated from experimental w data, at various temperatures.*

	B				
T_{abs}	A	Kr	N_2	T_{abs}	B AIR (LANGMUIR)
1050	3.1 mm	1.6 mm	3.0 mm	1100	4.2 mm
1315	2.3	1.3	4.3	1300	4.9
1537	3.3	0.7	4.2	1500	4.9
1735	3.6	1.1	5.0	1700	4.7
1910	4.2	2.1	5.1	1900	3.8
	3.3	1.4	4.3		4.5

With this equation we can convert measurements with any a to a common basis. In Table II we compared the B values derived from Table I with those found by Langmuir in air.

In view of the unsystematic temperature dependence of the B values both in Langmuir's and in our experiments, the average values in N_2 and in air agree tolerably well. Neither Langmuir nor we dared to conclude on this basis that B rises with the temperature, although the data of Tables I and II seem to indicate that this is the case.

A New Conventional Definition of b and its Direct Determinations

If Langmuir's hypothesis, that practically the whole temperature difference lies on a relatively thin gas film, is true, then two parallel, heated filaments should not influence each other's watt

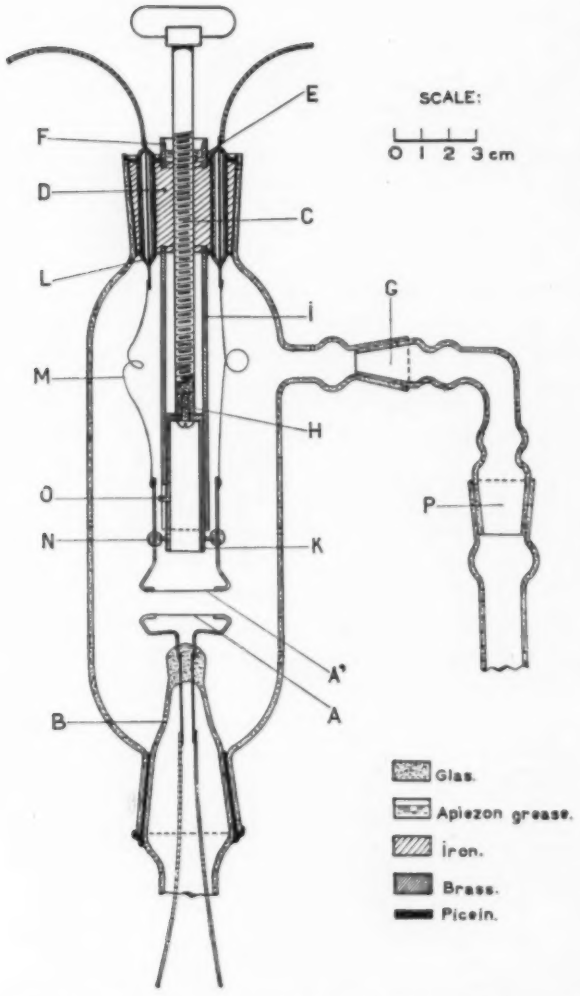


FIG. 1. Apparatus for determining the distance of thermal interaction between incandescent filaments.

consumption at a distance greater than the thickness of this layer. We assumed that they begin to influence each other rather abruptly when they approach to a smaller distance and our experiments verified this assumption. We now *define* the diameter of the film as the distance at which the thermic interaction of two parallel filaments can be detected if the filaments are capable of being moved towards each other.[3] Naturally this definition involves a standard method of detecting the interaction. This means, that the diameter of the "film" will increase with increasing precision of our wattage measurements, but this cannot be otherwise as long as we persist in speaking of the limit of a film, where evidently no such abrupt limit exists.

One type of our *apparatus* which allowed a heated filament to move parallel to an other one is shown in Fig. 1. A and A' are the tungsten filaments about 35 mm long, welded to the flattened nickel electrodes. Part B, the glass holder of the immobile filament A, is cemented with Picein into the tube. The mobile filament A' moves parallel to itself if the micrometer screw C (pitch 0.5 mm) is turned around. The metal cone D must be carefully cemented with Picein into the outer glass tube and adjusted in such manner that A and A' shall be in contact along their total length. The winding E must be cut with utmost precision to hold a high vacuum in the tube against the atmospheric pressure. The winding moves in the reservoir F which contains a mixture of Apiezon grease N and L, C must be screwed up and down in the grease a few times to ensure a reliable vacuum and the grease has to be remelted often. A second glass joint P, perpendicular to G, permits the tube to turn in a second direction so that the filaments can be brought into any desired position.

Part K of the apparatus received its translatory movement from the micrometer C by means of the loose screw H and moved in the exactly fitting brass cylinder I. The little rod O moved in a slit in the side of I and kept K from turning around its axis. Distance measurements with the micrometer were read always from the same direction to eliminate the end play of about 0.2 mm. The electric leads pass the conus D through two holes; they are insulated by small glass tubes L and are held vacuum tight by Picein. Since the leads within the tube must be flexible to allow for the movement of K, they are made of parallel threads of 0.05-mm copper wire M. The electric circuit of A' is insulated from K by the glass pearls N. The lead-in wires of A' had a resistance of about 0.12Ω, those of A about 0.05Ω; correction was made for these resistances when necessary.

The measurement of the "film"

The tube was filled with gas of desired composition and pressure and the plane of filaments AA' adjusted either in horizontal or vertical

[3] A similar method was evolved, independently of us, in the Philips Laboratories. See W. Geiss, Philips Techn. Rundschau. 1, 99 (1936).

position. The two filaments were connected in series to each other and to a variable series resistance to a d.c. source of 120 v. The temperature of the middle part of the filaments was read on an optical micropyrometer. Current and voltage on the filaments was read always on the same type of Siemens instrument (10Ω, 45 mv and 1000Ω, 3 v, respectively, 150 scale intervals) with appropriate shunts and series resistances, respectively, so that the relative sensitivity of the measurements was always nearly the same. By this circuit the series resistance held the current practically constant; if the filament A' was screwed near enough to A to affect each others temperature, this manifested itself by increasing the resistance of the filaments and the voltmeter began to move towards increasing voltage. The distance between A and A' where this movement was seen to begin is our conventional diameter of the Langmuir film (b). Thus the measurement of this film is reduced to a micrometer reading and can be performed within a minute with a reproducibility of at least 15 percent. The measurements were always repeated six times to give a reliable mean value.

The diameter of the gas film in nitrogen

The original method of Langmuir left open the question, whether b depends on the *temperature* or not. Our measurements leave no doubt that b increases with the temperature, in first approximation linearly. Of course there cannot be any film when the filaments are at room temperature; the whole gas is then at rest so that an extrapolation of the results to room temperature is not permitted. Fig. 2 shows the temperature dependence of b in nitrogen of atmospheric pressure for the filaments standing in a vertical (1) and in a horizontal plane (2). Evidently the gas temperature and convection can only have rotational symmetry around the vertical filaments and there is little sense in speaking of the "diameter" of the gas film around a horizontal filament (see Fig. 12). But the distance of thermic interaction can be measured also in this case and means the horizontal diameter of the noncircular cross section of the film. *The pressure dependence of b* around wires both vertical and horizontal is represented on Fig. 3. Here again the film has only rotational symmetry around the vertical wires. The general trend of the curves agrees with Langmuir's results.[4]

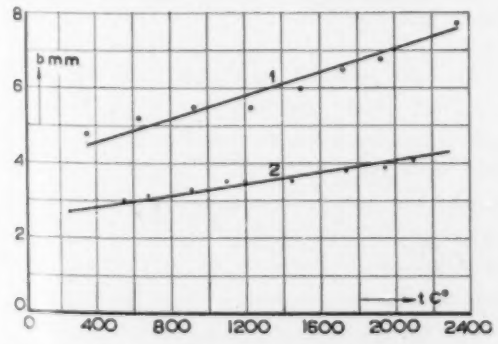


FIG. 2. Film diameter (= of interaction) as a function of filament temperature in N_2. Pressure = 1 atmos.; filament diameter = 0.099 mm. 1, vertical and 2, horizontal filaments.

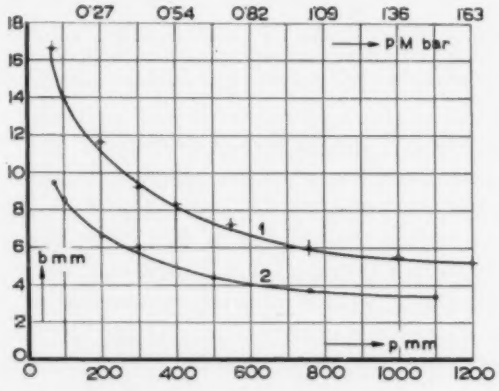


FIG. 3. Film diameter as a function of gas pressure in N_2. Temperature = 1500°C; filament diameter = 0.099 mm. 1, vertical and 2, horizontal filaments.

The dependence of b from the diameter of the wire

The experiments were made with vertical, 40-mm long, tungsten wires in nitrogen of 760 mm (1.04M bar) pressure, at 1500°C and 1220°C, respectively. The diameters of the wires were 9.3, 25.8, 40.8, 99 and 144 microns. It was extremely difficult to keep the 9.3-μ wire straight without stretching it so far that it failed at high temperatures; besides they had a great tendency to form "spots" and burn out. The measurements with this thin wire may be in error to within ±10 percent, whereas the rest is probably correct within ±5 percent. Some results are

[4] I. Langmuir, Proc. A. I. E. E. **31**, 1011 (1912); "Recalculation of Kennelly's experiments." See: Kennelly, Trans. A. I. E. E. **28**, 363 (1909).

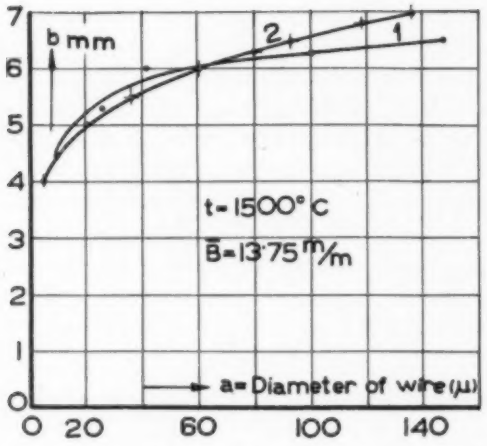


FIG. 4. Film diameter as a function of filament diameter in nitrogen at 1 atmos. and 1500°C. (Curve 1.) Curve 2 is calculated using Langmuir's equation.

summarized in Fig. 4. Between 40 and 140μ there is hardly any influence of the wire diameter on the thickness of the layer. The data can be used to test the validity of Eq. (2) in the same manner as in Table IV of Langmuir.[5] We calculated the B values for each experimental point and found as mean values 13.8 ± 1.1 mm at 1500°C and 12.3 ± 0.7 mm at 1200°C; the mean percentual standard deviation thus amounts to 7 percent. We also calculated the mean standard deviation from Langmuir's results at various temperatures and found it to be 18 percent. Farther on we reconstrued the graph analog to Fig. 4 from Langmuir's B values which fitted somewhat worse to their own theoretical curve than our curve 1 fits to 2. Thus the validity of Eq. (2) would seem to be assured with a greater precision than in Langmuir's original paper. But a glance at Fig. 4 (and similar curves, not published) shows that the deviation of the experimental points from curve 2, which was calculated from the equation using the mean values of our own experimental B's, is probably systematic. We think that Eq. (2) is a good approximation of the facts, especially in our range of wire diameters, but that it must have a fundamental deficiency and is not apt to represent the precise functional dependence between a and b. We note that Langmuir used horizontal wires, so that the absolute value of his b's is about half of ours. We also made some experiments with horizontal wires and arrived at b values similar to Langmuir's. However, we reproduce only vertical-wire results because of the rotational symmetry of this arrangement. Some experiments with different wires at various pressures indicated that the difference of the b values for different wire diameters decreases with decreasing pressure. Ch. W. Rice[6] derived a dimensional formula (see later on: Eq. (5)) where, with his experimental constants for vertical wires, a cancels out. That is, b should be independent of the diameter of the wire. Our results stand midway between Langmuir's and Rice's formula.

[5] I. Langmuir, Phys. Rev. **34**, 416 (1912)

The temperature and pressure dependence of b in different gases

We examined the five rare gases, nitrogen and hydrogen. The rare gases were freed even from traces of not rare gases by evaporating barium metal in their containers. The argon contained

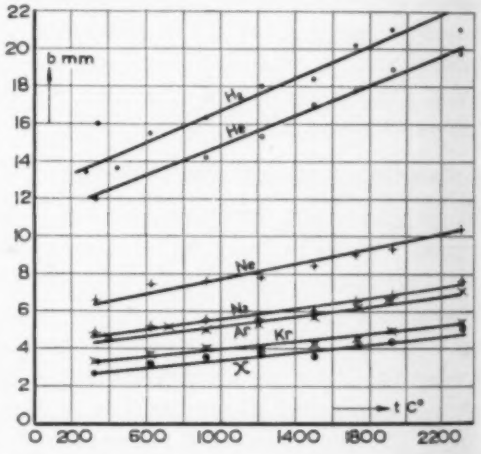


FIG. 5. Film diameter as a function of temperature in various gases at 1 atmos. pressure and filament diameter of 0.099 mm.

about two percent nitrogen and the krypton contained about five percent xenon. Helium and neon may have been contaminated with each other and the xenon with krypton to a small extent. The nitrogen and hydrogen were practically pure. As will be shown later on (see Fig. 11) the behavior of gas mixtures lies midway between those of the components so that the few percents of adjacent gases could not materially affect our results. The expensive gases, helium, neon and xenon were transported by means of a mercury

[6] Ch. W. Rice, reference 1, p. 1291.

piston from their containers to the apparatus and back to the containers.

The results of our measurements at *different filament temperatures* are summarized in Fig. 5. The straight lines reveal that b increases linearly with the temperature. But besides this linearity they are governed by a second rule, that the slope of the highest line (H_2) is the steepest, that of the lowest one (X) the least steep, and so on. We found that this rule is a simple quantitative one: the slopes for different gases are proportional to the b values at a given temperature (e.g. 1000°C). The lines on Fig. 5 are all drawn according to the equation

$$b_t = b_{1000}(1 + /t - 1000/.0{,}000255) \qquad (3)$$

and the experimental points fit these lines excellently. Langmuir derived his b values from wattage measurements by Eq. (1). To arrive at a rough approximation of the temperature function we put $\int_0^T K dT$ proportional to T^1 according to simple kineteic theory

$$\ln \frac{b}{a} \sim \frac{T^1 - 400^1}{w}. \qquad (4)$$

We found in our experiments that w increases between 1000° and 1400°K about in proportion

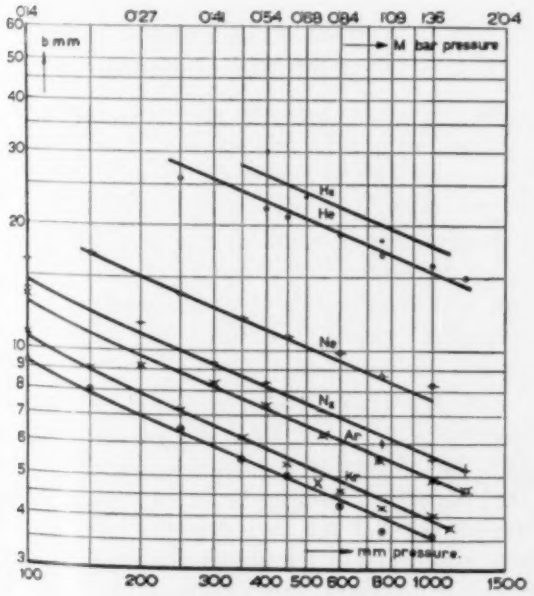


FIG. 6. Film diameter as a function of the pressure of various gases, logarithmically. Filament temperature 1500°C, diameter 0.099 mm.

with T but gradually becomes proportional to $T^{1.8}$ as the temperature rises to 2200°K. The experimentally determined dependence of w on T thus yields, by Langmuir's method of calculation,

TABLE III. *Film diameter at atmospheric pressure around a vertical filament of 99μ φ.*

GAS	$b_{700}(b_{1.01M}$ bar)
H_2	20.5 mm
He	17.5
Ne	8.6
N_2	6.3
A	5.6
Kr	4.4
X	3.9

at first a slightly increasing, then a constant, and finally a slightly decreasing movement of b with T. As a matter of fact this is the case in Langmuir's Table IX. Instead of accepting the improbable result that b can fall with rising temperature at high filament temperatures, Langmuir concluded that the deviations from a constant b were eventually caused by experimental errors and that b was independent of T in first approximation. We prove later on that our b values are probably diameters of the 365°K isotherms around incandescent filaments and thus represent the same kind of quantity as in Langmuir's definition. The fact that our direct b values increase with the temperature, although by Eq. (4) and the experimental wattage values they should pass through a maximum, clearly shows that at higher temperatures an increasing part of the energy is carried away by convection and not only by conduction. Ch. W. Rice[6] derived a formula for b by dimensional consideration and calculated the necessary constants from experimental data. For long vertical cylinder he finds

$$b = 8.65a(\eta/\rho\alpha g)^{\frac{1}{4}} 1/\Delta t^{\frac{1}{4}}, \qquad (5)$$

where η, ρ, α stand for the viscosity, density, thermal expansion coefficient of the gas and g for the acceleration of gravity. If this formula holds, the film thickness should decrease with rising temperature difference between filament and ambient gas. Our experiments definitely rule out this possibility and thus the validity of Eq. (5) is disproved.

We examined further on the *effect of pressure on the diameter of the gas film* in our seven different

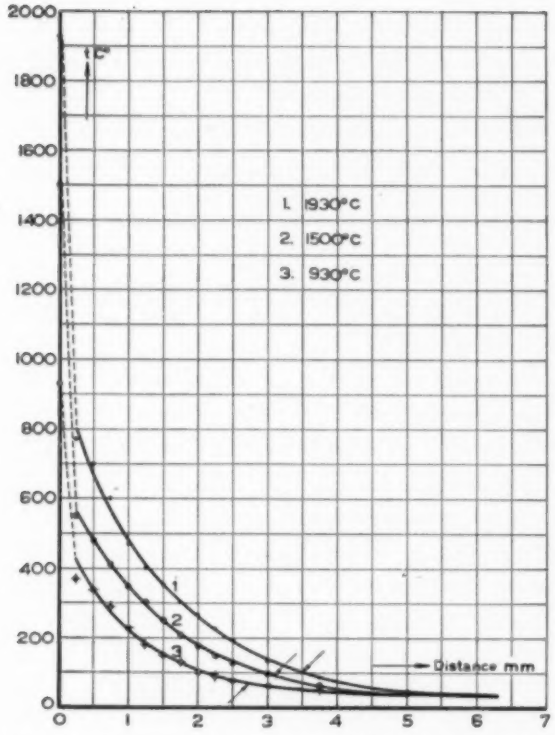


FIG. 7. Gas temperature as a function of distance from filament at different filament temperatures and 0.099 mm diameter. 1 atmos. N_2.

gases at a filament temperature of 1500°C. The measurements on hydrogen are rather uncertain because they had to be made in a hurry; diffusion of air through the micrometer winding would have caused an explosion. The results are plotted as log b against log p in Fig. 6. We find that b increases very quickly with decreasing gas pressure in the manner indicated on Fig. 3 for N_2. The experiments, however, could not be extended to pressures above 1200 mm (1.64M bar) so that it can not be decided, whether b eventually tends towards an asymptotic value in the interesting region of higher pressures or not. Within the range of our experiments we can approximate our results with the nonasymptotic formula:

$$b/b_1=(p_1/p)^{0.42}. \qquad (6)$$

The logarithmic straight lines are drawn with this slope (−0.42) in Fig. 6 and the b values corresponding to one atmos. pressure are given in Table III. The exponent of B from our b data is −0.56, which definitely disagrees with the one mentioned in Langmuir's paper −0.75. It must not be forgotten, however, that our definition of the layer diameter is not necessarily identical with Langmuir's original definition. The more detailed dimensional formula of Rice (Eq. (5)) with his constants for vertical filaments yields −0.67 for the exponent of p (p being proportional to ρ). This is only a little nearer to our experimental result than Langmuir's exponent.

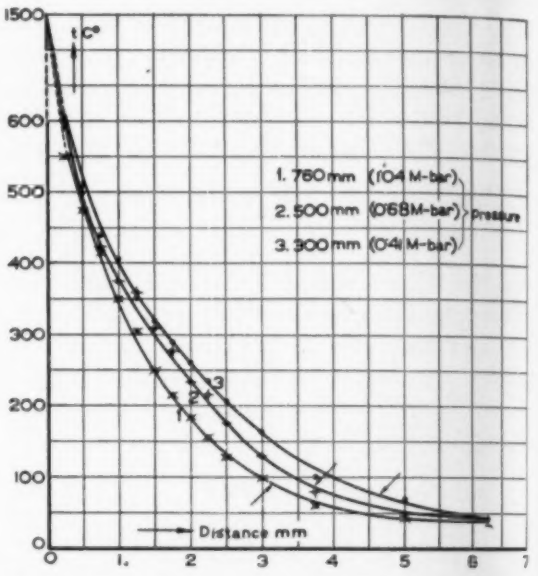


FIG. 8. Gas temperature as a function of distance from filament at different N_2 pressures. Filament temperature 1500°C, diameter 0.099 mm.

THE TEMPERATURE FIELD AROUND INCANDESCENT FILAMENTS

All our results indicate, that the conception of a "film" around an incandescent filament is not quite sufficient to represent the facts. The limit of the film depends on our definition and, although some remarkably simple rules could be found governing the diameter of this arbitrary film, we thought it fit to examine thoroughly an accessible physical entity, the temperature around the hot filaments.

The apparatus was essentially the same as shown in Fig. 1. The hot filament, about 40 mm long, was welded to the standing electrodes (A). The resistance thermometer was a tungsten filament of 9.3 microns diameter and 10 mm long, supported by the mobile electrodes (A') and adjusted opposite the uniformly incandescent upper middle part of the hot filament. We first aged the filament by heating it electrically to

about 1500°K for a minute. The thermoelectromotive forces on both ends of the thin filament did not allow us to measure the resistance in a bridge-circuit, so that we measured the voltage drop caused on the filament by a current of 0.5 ma; the thermovoltage was eliminated by commutation. The resistance of the leads was taken into account. We calculated the temperatures from the resistance ratio by tables of Jones-Langmuir.[7] It is, of course, always a

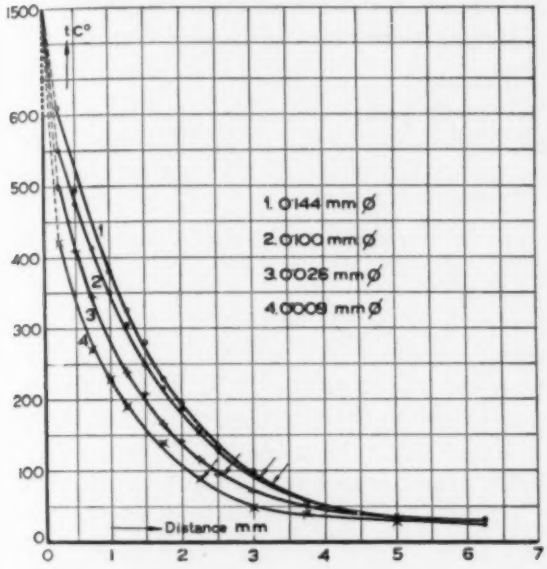


FIG. 9. Gas temperature as function of distance from filament at varying filament diameters. Temperature =1500°C; gas pressure = 1 atmos. N_2.

delicate question, whether temperature measurements in a moving gas with locally different temperatures are real or not. All thermometers, thermocouples or resistance wires disturb the stream lines of the flowing gas and cause a loss of heat by conduction. We measured temperature fields with resistance wires of decreasing diameter down to the said 9.3μ and found practically no difference between 100- and 9.3-μ wires at distances greater than one mm from the hot filament in nitrogen. At a distance of 0.5 mm the difference was 50° between the 100-μ and 9-μ wires, but only amounted to 10° between the resistance wires of 9μ and 26μ diameter. A comparison of the measurements with a copper-constantan thermocouple in hydrogen gave good concordance (within 10°) of the two methods. In nitrogen, however, where the temperature gradient and thus the conduction by the leads is greater, the temperature readings on the thermoelement were substantially too low. We changed the thermocouple, taking a Pt-Pt Rh one (35 microns), and could reduce the difference to about −30° in the region of one mm distance from the hot wire. This thermocouple was used, however, only to measure the detailed temperature field around a vertical filament (see Fig. 13) where it was absolutely necessary to measure with a point-like instrument.

Figure 7 represents the temperature drop around vertical 0.099-mm tungsten wires in nitrogen at temperatures 930°, 1500° and 1930°C, respectively. The nearest points are probably too low; the curves are drawn above them. It is interesting to note that the greater part of the total temperature drop falls within a distance of 0.25 mm and could not be determined. The pressure dependence of the field around vertical filaments of 1500°C is shown in Fig. 8 for 760,500 and 300 mm (1.04, 0.68, 0.41M bar) pressure, whereas the influence of the filament diameter may be seen on Fig. 9. Fig. 10 contains the

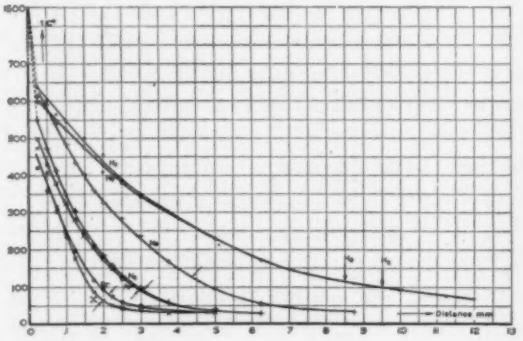


FIG. 10. Gas temperature as a function of distance from filament in different gases at 1500°, 1 atmos. and filament diameter of 0.099 mm.

temperature measurements around 0.099-mm filaments of 1500°C in the seven different gases mentioned above, at atmospheric pressure.

Figure 11 shows the horizontal temperature drop beside a horizontal filament of 0.099 mm ϕ at 1500°C in hydrogen, nitrogen and a 50 percent mixture of these gases at atmospheric pressure. The temperature curve of the mixture is about halfway between its components and falls monot-

[7] H. A. Jones and I. Langmuir, Gen. Elec. Rev. 30, 310 (1927).

onously and smoothly, just as if it were in a pure gas.

The corresponding b values of the preceding chapters are indicated by arrows on the curves. We see that these previously determined b values correspond to an average temperature of 90°C

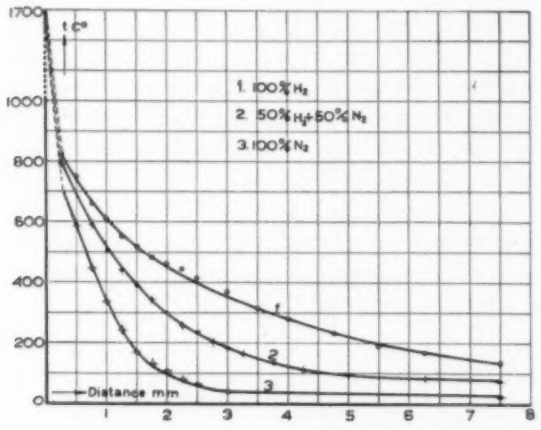


FIG. 11. Gas temperature as a function of distance from filament of 0.099 mm ϕ and 1700°C in 100 percent H_2, 50 percent H_2+50 percent N_2 and 100 percent N_2 of atmos. pressure.

on our 14 curves in Figs. 8–11. Only four intersections are outside the interval of 80–100°C. Neon shows the highest point of intersection, 125°, but the remarkable fact that the neon curve (Fig. 10) also cuts the He curve at high temperatures makes it probable that the temperature measurements in neon are systematically too high. (The distance at which the neon curve sinks to 90°C is 17 percent greater than the previously determined film radius $b/2$.) We calculated the standard deviation of the difference of the $b/2$ values and the distance where our 14 curves cut the 90°C isotherm in percents of the $b/2$ values and found (including helium and neon) ± 8.8 percent. In view of the fact that the differences are not systematic and this standard deviation is tolerably low, we think we are justified in concluding that our arbitrary definition of b corresponds to the distance at which the temperature of the gas has sunk to 90°C (say 365°K). Langmuir's original definition placed the limit of the film at the also arbitrary temperature of 400°K. Our temperature measurements in nitrogen (1500°C; one atmos.) showed (Fig. 7) that this temperature of 400°K is at a distance of 2.5 mm from the vertical filament, corresponding to a b value of 5.0 mm. The same distance from a horizontal filament is 2.25 mm and thus $b=4.5$ mm (Fig. 12). Langmuir's b value derived from watt measurements in air may be calculated from his B constant at 1700°K (4.7 mm) for the same filament ($\phi=0.10$ mm) and yields 2.8 mm. The difference indicates that his concept of the standing gas film does not represent the facts in an adequate manner.

Measurements of the detailed temperature field around horizontal and vertical filaments

It seemed certain that there *is* a vertical gas movement beside incandescent filaments. This upward streaming gas must needs cause a distortion of the temperature field. We therefore

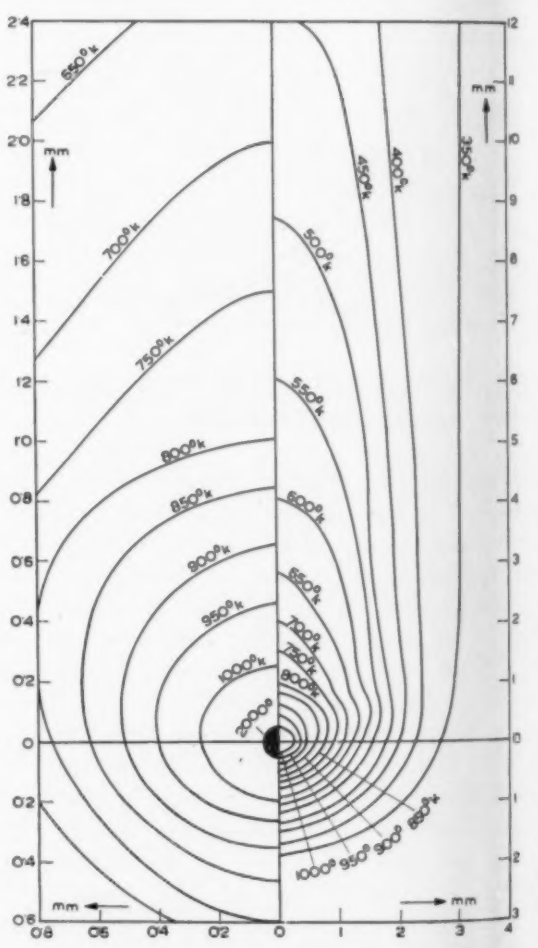


FIG. 12. Isothermals around a horizontal filament of 0.099 mm ϕ at 2000°K in 1 atmos. N_2. Vertical section. Left side is a 1 : 5 magnification of the right side.

determined the temperature in a vertical section all around a horizontal filament with our resistance thermometer (Fig. 12) and in a vertical section beside a vertical filament with the Pt-Pt Rh thermocouple mentioned above (Fig. 13). The measurements were performed in a glass bulb of 20 cm ϕ having an opening on the top and another on one side. Through these openings a vertical and a horizontal micrometric apparatus (analogous to Fig. 1) was adjusted with Picein so that the incandescent filament could be moved up and down, and the thermometer in a horizontal direction. Thus the thermometer could be brought in any desired relative position to the filament. The figures leave no doubt about the great role of convection besides conduction: The isothermals are "blown upward" along the vertical axis and are far from the rotational and planar symmetry, respectively, that would be the case for pure conduction.

The upward gas stream also cools the lower end of a vertical filament and heats its upper end; this asymmetric temperature distribution was determined by means of an optical micropyrometer on the filament of Fig. 13 and is represented on Fig. 14. The temperature of dull red heat of course limits these measurements.

These results are in agreement with the experiments of B. Ray and S. Ch. Pramanik,[8] who measured the temperature field around a warm cylinder of 14 mm ϕ in air at 223°C and took "Schlieren" method photographs of gases around incandescent wires. The "Schlieren" method was also used by O. A. Saunders and M. Fishenden[9] and R. Hermann[10] while W. Weizsäcker[11] measured the temperature around horizontal cylinders which were heated only slightly above the temperature of the enclosing vessel.

In view of the temperature distribution around incandescent filaments (Figs. 12 and 13) there can be hardly any doubt, that there *is* a convection in their neighborhood, even within the borders of the "layer." To make quite sure, we observed the upward drift by means of a minute silica-glass anemometer (a thin 1×1 mm silica splinter attached to a thin, horizontal silica filament). The upward deviation of the splinter increased as it was brought nearer to the filament and the deviation was greatest just before the anemometer effectively reached it. This agrees well with the experiments of Lax and Pirani,[12] who made a kinematographic study of MoO_3 smoke moving upwards in the vicinity of an incandescent tungsten coil in argon-nitrogen atmosphere. Vertical streaming velocities up to 100 cm/sec. were detected in this manner in a 50-v 750-w lamp. (35 cm/sec. in a 220-v 40-w lamp.)

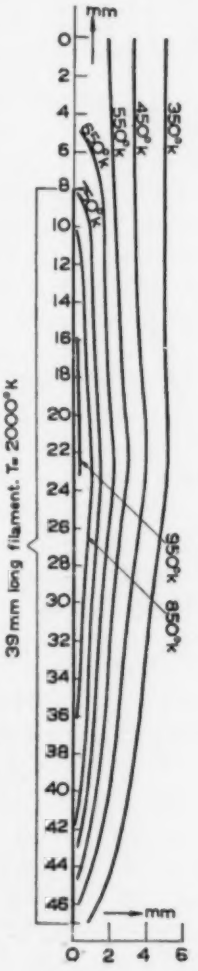


FIG. 13. Isothermals beside a vertical 0.099 mm ϕ filament of 2000°K in 1 atmos. N_2. Vertical section.

[8] B. Ray, Proc. Ind. Assoc. for Cult. of Science **6**, 95 (1920/21), page 105; S. Ch. Pramanik, Proc. Ind. Assoc. for Cult. of Science **7**, 115 (1922).
[9] O. A. Saunders and M. Fishenden, Engineering **139.II**, 483 (1935).
[10] R. Hermann, reference 2, pp. 13, 21.
[11] W. Weizsäcker, Physik. Zeits. **37**, 641 (1936).
[12] E. Lax and M. Pirani, Zeits. f. Physik **58**, 7 (1929).

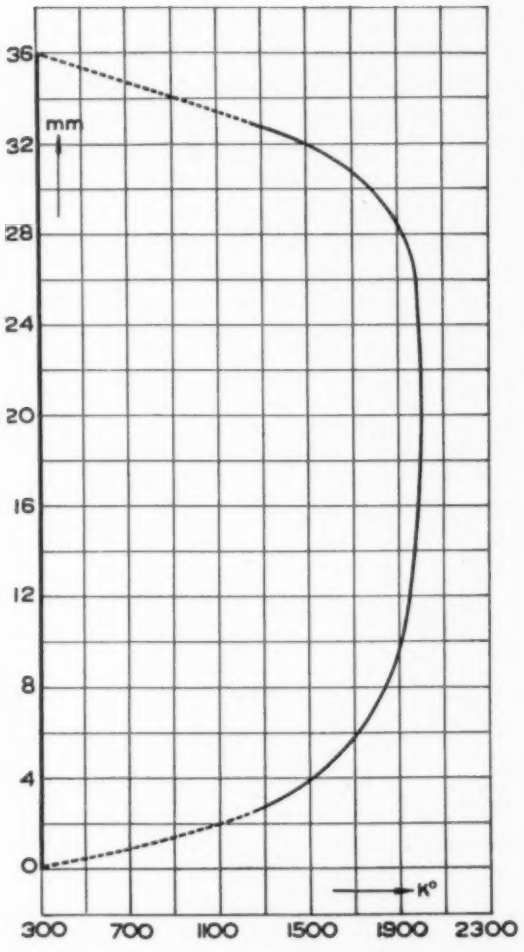


FIG. 14. Temperature distribution along the vertical filament of Fig. 13.

There is still another mechanical experiment which enables us to see the effect of upward gas stream on the hot body itself. We fastened a 110-v 100-dekalumen tungsten coiled coil to a horizontal silica glass filament about 10 cm long and 0.2 mm strong. The pointed edge of the silica could be followed with a horizontal microscope and we found that the limit of sensitivity of this balance was about 0.1 mg. By burning the coil in an atmosphere of argon the gas stream visibly lifts it upwards with an upward force of one mg. In one fortuitous case the balance not only rose after the current was turned on but began to swing up and down with a period of about one second and the amplitude of this oscillation was about 5 mm. By reducing the pressure to 380 mm (0.52M bar) the oscillation disappeared. This very interesting phenomenon could not be reproduced a second time after the silica filament had broken. It must have been a haphazard resonance of the mechanical system with the periodic formation of eddies in the hot gas on the surface of the incandescent coil. This process is known to occur in gas-filled incandescent lamps and it generally has a period of the aforesaid order of magnitude. We observed it in a compensation circuit as an oscillation of current (periodic cooling of the coil) and found that it was strictly periodic in the case when the balance oscillated mechanically, but gave only an average period of one sec. on other balances, which did not oscillate. There seemed to be a mutual influence of mechanical and hydrodynamical movements with a strictly periodic eddy formation as a result. Now the rise of the coil as well as the existence of the well-known current beats (eddy formation) in gas-filled lamps definitely prove that the gas streams upward in the immediate neighborhood of the coils and filaments.

Dr. G. Vészi devised an interesting experiment in this connection. The upward gas stream can be partly compensated by blowing gas downwards from above the filament. As a matter of fact the watt consumption goes through a minimum with the wind velocity, because the wind at first only counteracts the upward convection and thus reduces the convective losses. At greater wind velocities, however, the gas current turns downwards and cools the filament again, increasing the wattage. The existence of the minimum is a proof for the existence of the upward convection.

THE WATT LOSSES BY CONVECTION AND CONDUCTION

It is interesting to compare the watt consumption for maintaining e.g. the temperature at 1500°C in the different gases. A vertical, 0.10-mm tungsten filament of 35-mm length loses 1.0 watt by radiation and by conduction of the leads. Subtracting this value we arrive at the wattage loss caused by the different gases. These values (W) are tabulated in Table IV together with the thickness of the "gas layer" (b) at atmospheric pressure and with some characteristic constants of the gases.

It is interesting that while the temperature distribution is nearly identical in H_2 and He (Fig. 10) the watt consumption in H_2 is 27.1 against 18.9 in He. The case is quite similar for the pair N_2-A (4.8 w : 3.5 w). This means that heat must flow at a much greater rate across the diatomic gases to produce the same temperature gradient as in the monoatomic ones: the two rotatory degrees of freedom play their part in conduction. However it must be attributed to mere chance that the constants that determine conduction *and* convection in gases act just so that the temperature field of the mentioned pair of gases is nearly identical for the same filament temperature.

Assuming pure conduction, Eq. (1) correlates any distance from the filament (x) with the temperature at that distance (T_x), the wattage per cm filament (w) and the filament temperature being given. Eq. (1) may be rewritten in this sense:

$$\ln \frac{x}{a} = 2\pi \int_{T_x}^{T} K dT \Big/ w. \qquad (7)$$

We may use this equation to calculate the $x-T$ curve for a given experimental wattage or vice versa. *Experimentally*, after correcting for radiation and end losses, we found $w=7.8$ watt/cm in H_2 and $w=1.4$ watt/cm in N_2 for atmospheric pressure, $T=1500°C$ and $a=0.099$ mm. On the other hand, we calculated w from Eq. (7) by substituting x for the distances read from the temperature-distance curves (Fig. 10) for $T_x=400°K$ (8.0 mm for H_2 and 2.6 mm for N_2), and arrived at 7.1 watt/cm and 1.23 watt/cm, respectively. The difference, that is roughly only 10 percent of the wattage, must have been carried away by convection; this shows that regarding wattage the theory of pure conduction gives fairly good results.

But the difference between theory and experiment is much greater if we use Eq. (7) the other way, i.e., to calculate the distance at which the temperature should be 400°K if the experimental wattage were lost only by conduction. We find 5.7 mm for H_2 and 2.0 mm for N_2, that is, the experimental distance exceeds the theoretical one by about 35 percent.

TABLE IV. *Watts lost and b values around 0.1 mm φ filament and physical constants of the seven gases used in our experiments.*

Gas	$W_{1500°}$	$b_{1500°}$	Viscosity $\cdot 10^7$ at 20°C	Sutherlands C	Heat conductivity $\cdot 10^7$	Molar heat C_v	Molar weight M
H_2	27.1	19.0 mm	876	85	4130	5.57	2.02
He	18.9	17.0 "	1941	75	3363	3.00	4.00
Ne	7.1	8.7 "	3111	55	1092	3.00	20.18
N_2	4.8	6.2 "	1818	115	566	4.96	28.01
A	3.5	6.0 "	2217	160	406	3.00	39.94
Kr	2.1	4.5 "	2400	190	190	3.00	83.7
X	1.6	4.0 "	2260	250	124	3.00	131.3

Conclusions

We hope to have given sufficient evidence that not only is there no stationary film of mm dimensions around an incandescent wire, but that the velocity of convection increases up to the inner limit of our measurements ($\frac{1}{4}$ mm) as we approach the filament. This seems to be the opinion of Rice also[13] who places the boundary of the film at the surface where laminar upward motion goes over in turbulance and not where the gas ceases to "stagnate." The only decisive experiment would be in this connection a systematic kinematographic study of gas movement around the filament, similar to the experiments of Lax and Pirani.[12]

The original theory of Langmuir, as given by Eq. (7) is not, however, *principially* adequate to describe the temperature field around an incandescent wire. For finite values of a, W and the integral, we always arrive at a functional dependence between x and T_x as given in the qualitative curve 2 in Fig. 15, and never at an experimental temperature-distance curve like curve 1. There is no place for a singular behavior of this function at room (and vessel) temperature; the curve goes right through this range and attains the absolute zero at a finite distance x_0:

$$\ln x_0 = \left(2\pi \int_0^T K dT \Big/ w\right) + \ln a. \qquad (8)$$

It would be perhaps natural to try to impose the boundary condition $T \to$ room temperature for $x \to \infty$ on Eq. (7), but it is impossible to satisfy the equation in this manner for any value

[13] Ch. W. Rice, reference 1, p. 1289.

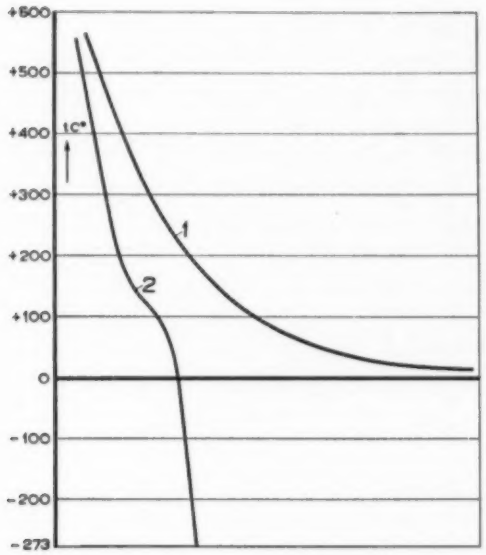


FIG. 15. Types of gas temperature-distance curves as found in experiments (1) and as calculated by Langmuir's equation of pure conduction (2).

of $w>0$. Thus the equation of pure conduction cannot be applied to the case of cylindrical symmetry; evidently the effect of the ends cannot be safely neglected. The case of spherical symmetry, however, is quite different. Integration of the same differential equation that yields Eq. (1) and (7) then yields:

$$\frac{1}{x_0}=\frac{4\pi}{w}\int_0^T KdT+\frac{1}{a}. \qquad (9)$$

We see that this equation can be satisfied by a finite value of w for $x_0\rightarrow\infty$. Evidently the spherical problem is physically real, there are no "end effects" to be neglected and thus it is natural that this case of pure conduction must be possible.

The reason that Langmuir's hypothesis worked well in the theory of convection seems to be that it was derived from and applied to wattage measurements only. We have seen that most of the energy is carried away by conduction. Probably phenomena of diffusion in the neighborhood of the filament may be also successfully treated with this working hypothesis, owing to the fact that diffusion as well as thermal conductivity is governed by the length of free path of the gas molecules. But the theory cannot be safely applied to the temperature distribution around incandescent filaments and the concept of a "stagnant" gas layer should be abandoned definitively. As a matter of fact, these "geometric" properties of the film around the filament were never examined up to now. The *film thickness* (b and B) in Langmuir's calculations was never checked by experiments, and only served to establish relationships between experimental *wattage values*. We conclude that the phenomena around an incandescent filament in a gas are composed from conduction and convection, the former accounting for the greatest part of wattage transfer, the latter giving rise to very pronounced gas movement and strongly distorting the temperature field against that of a hypothetical pure conduction.

SUMMARY

(1) The diameters of "stationary gas films" around incandescent wires were determined in A, Kr and N_2 by Langmuir's original method from wattage measurements. $B_A=3.3$ mm; $B_{Kr}=1.4$ mm; $B_{N_2}=4.3$ mm.

(2) The said diameter was redefined as the limit of thermal interaction between two parallel incandescent filaments. A direct method to measure this limit was evolved.

(3) The limit of interaction (diameter of "gas film") was determined for a series of gases around horizontal and vertical filaments as a function of filament diameter, temperature and gas pressure. Simple empirical relations were established between these variables and the limit of interaction.

(4) Temperature-distance curves were determined around vertical filaments under the same circumstances as under (3). It was established that our "limit of interaction" corresponds about to the 365°K isotherm.

(5) Detailed temperature distribution charts were determined around horizontal and vertical filaments.

(6) Evidence was gathered that no "stationary gas film" exists around incandescent filaments. It is demonstrated that the equation for pure conduction necessarily fails to describe the temperature field in the whole space around a warm infinite cylinder, whereas it gives sensible results around a warm sphere. The wattage transfer is, however, mainly caused by conduction.

Announcing...

Eastman Spectrum Analysis Plates

A NEW SPECTROSCOPIC PLATE especially designed for use in the metallurgical industries. It has higher contrast than usual process plates... very low background density... and adequate sensitivity. It is suitable for extremely rapid processing. Further information will be supplied on request.

Visit the Kodak Building at the New York World's Fair

EASTMAN KODAK COMPANY

Research Laboratories **ROCHESTER, N. Y.**

THREE DISTINCTIVE NEW BOOKS

TERRESTRIAL MAGNETISM AND ELECTRICITY

Edited by J. A. FLEMING, Carnegie Institution of Washington. 786 pages, 6¾ x 9¾. $8.00

This important new book is Volume VIII of Physics of the Earth, a series of monographs prepared under the direction of various committees of the National Research Council. In *Terrestrial Magnetism and Electricity* a group of distinguished specialists review the present status of our knowledge of the earth's magnetic and electric phenomena.

STATIC AND DYNAMIC ELECTRICITY

By WILLIAM R. SMYTHE, California Institute of Technology. *International Series in Physics.* Approximately 600 pages, 6 x 9. $6.00

In this book the author formulates the basic laws of electrostatics, magnetostatics, and electromagnetic theory, by concise vector methods, from the underlying experimental facts. The book gives an extended treatment of the mathematical technique for applying these laws to specific proplems, 100 of which have been worked out in the text.

ELECTRICITY AND MAGNETISM. An Introduction to the Mathematical Theory

By J. B. WHITEHEAD, Johns Hopkins University. *Electrical Engineering Texts.* 225 pages, 5½ x 8. $3.00

This book represents a compact development of the physical theories of electricity and magnetism beginning with the simplest manifestations of mechanical force by electrostatic and magnetic phenomena.

Send for copies on approval

McGRAW-HILL BOOK COMPANY, Inc.

330 West 42nd Street **New York, N. Y.**

Associates

THE American Institute of Physics gratefully acknowledges the support in 1939 of the following Associates of the Institute:

Agfa Ansco Corporation
Allegheny Steel Company
Aluminum Company of America
American Telephone and Telegraph Company
Bausch & Lomb Optical Company
Bell Telephone Laboratories
Crane Company
Dow Chemical Company
E. I. du Pont de Nemours & Company
Eastman Kodak Company
General Electric Company
General Motors Corporation
Gulf Research & Development Company
Libbey-Owens-Ford Glass Company
Loomis Laboratory
Massachusetts Institute of Technology, Department of Physics
Massachusetts Institute of Technology, Division of Industrial Cooperation
Radio Corporation of America
Research Corporation
Shell Development Company
Sperry Gyroscope Company, Inc.
Westinghouse Electric & Manufacturing Company

These Associates are contributing materially to the vigorous advance of pure and applied physics. A large part of their corporate dues is used to provide more pages for the recording of the results of research in the periodicals published by the Institute for the Founder Societies. Another part is devoted to the Institute's efforts to improve the opportunities and facilities for physical research in America through better public appreciation of the nature and importance of the science. The remainder helps provide for general work and special projects undertaken for the Founder Societies acting in cooperation through the Institute and designed to further those objects which the Societies have in common.

The American Institute of Physics cordially invites other interested corporations to become its Associates and will welcome their inquiries addressed to

The Director
American Institute of Physics
175 Fifth Avenue, New York, New York

IMPROVED "SPOTLIGHT" GALVANOMETERS

Rubicon **MULTIPLE REFLECTION** galvanometers have recently been improved through a redesign of the optical system. The definition of the line-image is now so sharp that readings can be estimated to .1 of a millimeter division.

These sturdy, self-contained galvanometers are available with sensitivities as high as .0006 microampere per millimeter. The scales, which are 100 mm. long, are remarkably proportional. Send for Bulletin 320.

RUBICON COMPANY

ELECTRICAL INSTRUMENT MAKERS

29 North Sixth Street Philadelphia, Pa.

FINE WIRES

TAYLOR PROCESS

THE TAYLOR PROCESS is a method for making wires from substances which lack ductility. The elements or alloys, because the drawing is done in glass or quartz, are in a very pure state. Lengths of more than a few feet cannot be obtained. The wires may be had with or without the glass insulation. We make Taylor Process wire of Pt, Rh, Au, Ag, Cu, Fe, Zn, Cd, Pb, Sb, Bi, Sn, Se, Te, Tl, Ga, and In, Constantan, Bi-Sn, Cd-Sb (used where large E.M.F. is desired) and many other alloys. Our wires are packed in containers holding one foot and, with few exceptions, the sizes are from one *mm* to one micron. We manufacture, too, fine wire by the Wollaston Method, by extrusion and by bare drawing.

WE ARE SPECIALISTS IN PLATINUM AND CAN SUPPLY IT IN WHATEVER FORM YOU REQUIRE

BAKER & CO., INC.

113 Astor Street, Newark, New Jersey

Linde

RARE GASES AND MIXTURES

NEON
ARGON
HELIUM
KRYPTON
XENON
MIXTURES

. . . highest purity
. . . for scientific purposes

Linde rare gases are of consistently high purity. They are used in the study of electrical discharges, in rectifying and stroboscopic devices, and in inert atmospheres where heat conduction must be reduced.

Linde rare gases are shipped in spherical glass bulbs, designed to facilitate removal without contamination. Special mixtures for experimental purposes can be supplied upon request.

The Linde Air Products Company

Unit of Union Carbide and Carbon Corporation

30 East 42nd Street UCC New York, N. Y. Offices in Principal Cities

The word "Linde" is a trade-mark.

STABILIZED A-C VOLTAGE

INPUT 95-130 volts
Instantaneous action
A magnetic unit

OUTPUT
115 volts ±1%
Stabilizes at any load within rating

No moving parts. Nothing to wear out.

RAYTHEON Voltage Regulator

APPLICATIONS Broadly it insures stable operation of all precision apparatus obtaining its power from an A.C. source, for example:—

- Insures constant brilliancy from all types of lamps.
- Improves the operation of X-ray equipment.
- Stabilizes all electronic apparatus.

Write for Bulletin 48-71G8P.

RAYTHEON MFG. CO. WALTHAM, MASS.

Standard or Experimental

RESISTORS

ALL TYPES · ALL SHAPES · ALL SIZES

Common resistor types made to uniformly high standards of quality . . . or special resistors for unusual applications . . . whatever the units you need IRC can supply them. The following bulletins detailing various IRC resistance types are available upon request:

☐ BULLETIN No. 1—Volume Controls and Potentiometers.

☐ BULLETIN No. 2—Metallized Type Resistors. Insulated ½, 1 and 2 Watts; High Frequency; High Range; High Frequency Power and High Voltage Power Resistors.

☐ BULLETIN No. 3—Insulated Wire Wound Resistors. Type BW from ½ to 1 and 2 Watts. Type MW 5 to 20 Watts, 6 std. sizes, any needed combinations.

☐ BULLETIN No. 4—Power Wire Wound Resistors. Types from 10 to 200 Watts, fixed and adjustable types, all mountings. Precision units, 14 types, 1/10 of 1% accuracy.

☐ BULLETIN No. 5—Attenuators.

☐ BULLETIN No. 6—Power Rheostats.

INTERNATIONAL RESISTANCE CO.

419 N. Broad St., Philadelphia, Pa.

IRC

INDEX TO ADVERTISERS

Name	*Page*
AJAX ELECTROTHERMIC CORP. ..... Ajax-Northrup high frequency electric furnaces from laboratory to plant sizes.	iii
BAKER & CO., INC. ..... Platinum crucibles, dishes, triangles, filter cones, anodes, cathodes, electrodes, platinum tipped crucible tongs, fine wires and bismuth foil.	vii
CENTRAL SCIENTIFIC COMPANY .....Cover 4 Manufacturers of Cenco Physical Apparatus and Instruments to meet all requirements of University, College and High School Physics Laboratories. Specializing in high vacuum pumps and development of instruments and apparatus for various sciences.	
EASTMAN KODAK COMPANY ..... Purified Organic Chemicals for research purposes; Plates for Photography, Photomicrography, Spectroscopy, Photometry, Astronomy; Wratten Light Filters; Cameras and Films.	v
THE EPPLEY LABORATORY, INC. ..... Standards of e.m.f. (standard cells). Precision electrical instruments; potentiometers, bridges, temperature bridges, volt boxes. Thermopiles and pyrheliometers.	ii
GENERAL ELECTRIC VAPOR LAMP CO. ..... Uviarc lamp for ultraviolet radiation. Sodium Lab-Arc Lamp.	iv
GENERAL RADIO COMPANY .....Cover 3 Manufacturers of electronic measuring instruments; vacuum-tube voltmeters, amplifiers and oscillators; wave analyzers, noise meters and analyzers, stroboscopes; laboratory standards of capacitance, inductance and frequency; impedance bridges, decade resistors and condensers; air condensers and variable inductors; rheostats, Variacs, transformers; other laboratory accessories.	

INDEX—*Continued*

Name	*Page*
INTERNATIONAL RESISTANCE COMPANY ..... Manufacturers of metallized and wirewound fixed and variable Resistors including high voltage and high frequency types.	viii
LEEDS & NORTHRUP COMPANY ..... Manufacturers of Galvanometers, Resistors, Bridges, Condensers, Inductances, Potentiometers, Testing Sets; Temperature Measuring, Recording and Controlling Apparatus; Instruments for Measuring and Controlling Conductivity of Electrolytes and Hydrogen Ion Concentrations.	i
LINDE AIR PRODUCTS CO. ..... Oxygen, Argon, Helium, Krypton, Neon, Xenon, Rare Gas Mixtures, Nitrogen, Hydrogen, Calcium Carbide, Acetylene, equipment for Oxy-Acetylene welding and cutting.	vii
MCGRAW-HILL BOOK CO., INC.	v
RAYTHEON MFG. CO. ..... Voltage stabilizers and regulators. Electrical measurement and control instruments.	vii
RCA MANUFACTURING CO., INC. ..... RCA Oscillators and Oscillographs, RCA Test Equipment, RCA Ultra-Sensitive DC Meter.	ii
RUBICON COMPANY ..... Galvanometers, electrometers, potentiometers, Wheatstone and Kelvin bridges, resistance boxes, hydrogen ion and conductivity apparatus.	vii